AY2018/2019 Semester 1 Conference (Week 13)

Sud, S. (2017,). Spiking deconvolution for seismic waves using the Fractional Fourier Transform. In SoutheastCon, 2017. IEEE.

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Signal Model

Recieved Seismic Signal in Discrete Time

$$s(i) = w(i) * r_e(i) + n(i)$$

$w(i)$ is the source seismic signal. $r_e(i)$ is the reflectivity function of the Earth, in terms of linear combination of impulse functions. $n(i)$ is the additive white Gaussian Noise (AWGN).

Assumptions

$w(i)$ is known or can be estimated, which will need $r_e(i)$ from the recieved signal $s(i)$, also known as the seismic trace. In practice, the multiple traces are conducted to have better accuracy for estimation.

Existing Method: Spiking Deconvolution (SD)

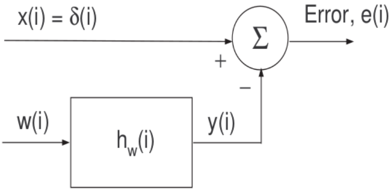

Fig. 1. Spiking Deconvolution as Wiener Filtering Problem

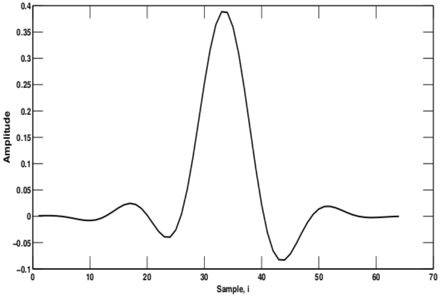

Fig. 2. Source Wavelet **w**

Existing Method: Spiking Deconvolution (SD)

Spiking Deconvolution (SD)

Estimation of the inverse of the source wavelet $w(i)$, denoted $h_w(i)$:

$$w(i) * h_w(i) \approx \delta(i) = \begin{cases} 1 & , i = 0 \\ 0 & , i \neq 0 \end{cases}$$

Minimise the error, $e = \frac{1}{N}\sum_{i=1}^{N} |\delta(i) - w(i) * h_w(i)|^2$, by letting $\frac{\partial e}{\partial h_w} = 0$, to determine the filter coefficients of the Wiener Filter $h_w(i)$. As the result, $\mathbf{h_w} = \mathbf{R^{-1}} \mathbf{r_{w\delta}}$, where $\mathbf{R} = \mathbf{w}\mathbf{w^H}$ is the covariance matrix of the input source wavelet and $\mathbf{r_{w\delta}} = \mathbf{w}\delta$ is the cross-correlation between the source wavelet and the delta. Earth Reflectivity: $\hat{\mathbf{r}}_e = deconv(\mathbf{s}, \mathbf{h_w})$. Or $\hat{\mathbf{r}}_e = xcorr(\mathbf{s}, \mathbf{w})$, cross-correlate the seismic signal with source wavelet.

Problem

Inverse of covariance matrix generally needs more samples beyond available. The deconvolution is not perfect, and causes instability.

Existing Method: Time Domain Deconvolution (TDD)

Time Domain Deconvolution (TDD)

$$\hat{\mathbf{r}}_e = deconv(\mathbf{s}, \mathbf{w})$$

Problem

As the result, the inaccuracies do occur, because the source wavelet $\mathbf{w}$ is not a delta function and affects the estimation of the reflectivity $\mathbf{r}_e$.

Fractional Fourier Transform (FrFT) Spiking Deconvolution

Discrete Time Fractional Fourier Transform (FrFT)

Let $\mathbf{X}_a, x \in \mathbb{R}^{N\times 1}$, and $\mathbf{F}^a \in \mathbb{R}^{N\times N}$, and $0 < |a| < 2$. Discrete Time FrFT:

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}$$

$$\mathbf{F}^a[m,n] = \sum_{k=0, k\neq (N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n]$$

$u_k[m]$ and $u_k[n]$ are the eigenvectors of $\mathbf{S} \in \mathbb{R}^{N\times N}$. $C_n = 2\cos\left(\frac{2\pi}{N}n\right) - 4$.

$$\mathbf{S} = \begin{pmatrix} C_0 & 1 & 0 & \dots & 0 \\ 1 & C_1 & 1 & \dots & 1 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \dots & \ddots & 1 \\ 1 & 0 & 0 & \dots & C_{N-1} \end{pmatrix} \in \mathbb{R}^{N\times N}$$

Algorithm of the Discrete FrFT Spiking Deconvolution

TABLE I

PROPOSED FRFT DOMAIN DECONVOLUTION (FRFT-DD) ALGORITHM

1. For $a = 0 : \Delta a : 2$ % Loop over all a

   $\mathbf{W}(a) = \mathbf{F}^{a}\mathbf{w}$; % Compute FrFT of $\mathbf{w}$

   $\mathbf{W}_{max}(a) = max(|\mathbf{W}(a)|)$; % Compute max value

   End

2. Find peak over all a to get the optimum

   $a_{opt} = \underset{a}{arg\ max}\ W_{max}(a)$;

3. Compute the FrFT, using a_{opt}, of the source wavelet and received seismic signal

   $\mathbf{W}(a_{opt}) = \mathbf{F}^{a_{opt}}\mathbf{w}$;

   $\mathbf{S}(a_{opt}) = \mathbf{F}^{a_{opt}}\mathbf{s}$;

4. Compute the estimated reflectivity function using deconvolution

   $\mathbf{R}_e(a_{opt}) = deconv(\mathbf{S}(a_{opt}), \mathbf{W}(a_{opt}))$;

5. Rotate back to the time domain

   $\hat{\mathbf{r}}_e = \mathbf{F}^{-a_{opt}}\mathbf{R}_e(a_{opt})$;

where the estimated reflectivity function is $\hat{\mathbf{r}}_e = \mathbf{F}^{-a_{opt}}\mathbf{R}_e(a_{opt})$

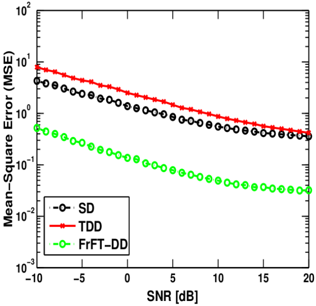

Fig. 3. $L = 3$ Length Reflectivity Function, $\mathbf{r}_e$

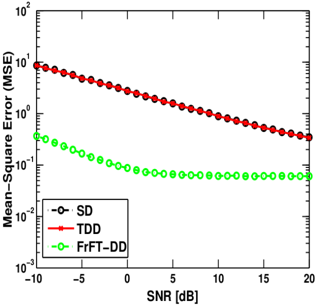

Fig. 4. $L = 7$ Length Reflectivity Function, $\mathbf{r}_e$

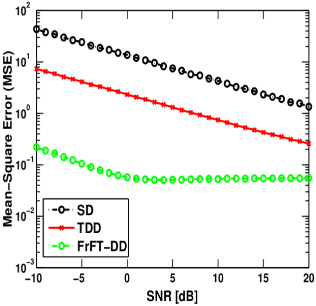

Fig. 5. $L = 11$ Length Reflectivity Function, $\mathbf{r}_e$

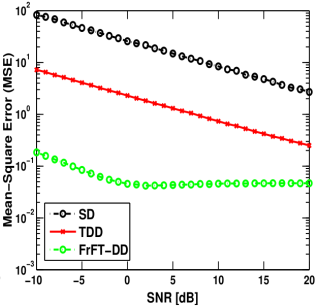

Fig. 6. $L = 15$ Length Reflectivity Function, $\mathbf{r}_e$

The End

Conclusion

Compare to the Time Domain Deconvolution (TDD), the discrete time FrFT spiking deconvolution reduces error and inaccuracies as the large sample support is not required. Also, the discrete time FrFT spiking deconvolution improves deconvolution accuracy by shrinking the source wavelet **w** to an impulse function (spike).

Future Work

Improve the proposed algorithm of the Discrete Time FrFT Spiking Deconvolution, by further reducing the Mean Square Error (MSE) and apply FrFT to other practial situations relevant to the research field of geoscience or remote sensing.