

Eikonal equation and its anisotropic description

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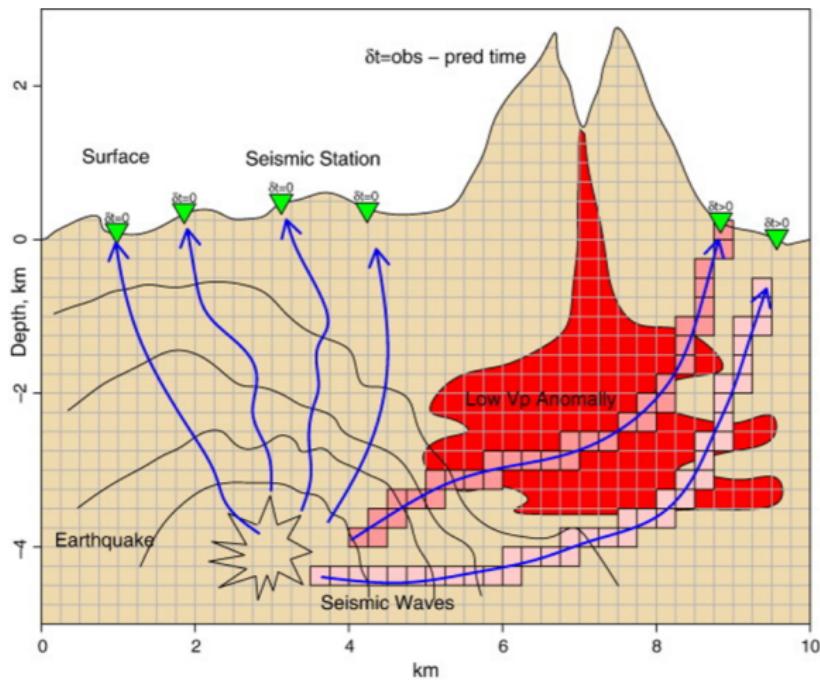
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Outline

1 Eikonal equation

What is eikonal equation

$$\frac{x_B - x_A}{T(x_B) - T(x_A)} \approx F(x_B)$$



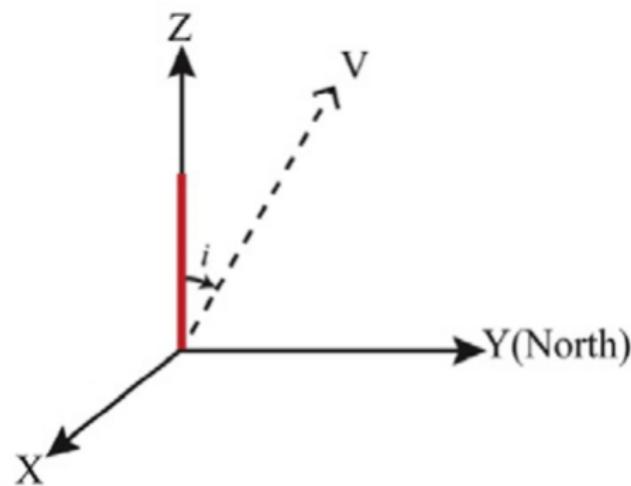
What is anisotropy

1 Isotropic Media.

$$|\nabla T|F(x) = 1,$$

1 Anisotropic Media.

$$|\nabla T|F(\nabla T|x) = 1.$$



How to determine "F"

Anisotropic Parameters

$$\alpha_0 = \bar{V}_P, \beta_0 = \bar{V}_S, \epsilon, \gamma, \delta \rightarrow F(\theta)$$

Table 1. Measured anisotropy in sedimentary rocks. This table compiles and condenses virtually all published data on anisotropy of sedimentary rocks, plus some related materials.

Sample	Conditions	V_p (m/s)	V_s (m/s)	ϵ	δ^a	δ	γ	ρ (g/cm ³)
Taylor's sandstone	$P_{eff} = 0$ MPa, saturated	11 050 3 368	6 000 1 829	0.110	-0.127	-0.035	0.255	2.500
Mesaverde (4903) ² mudshale	$P_{eff} = 27.58$ MPa saturated, undrained	14 860 4 529	8 869 2 703	0.034	0.250	0.211	0.046	2.520
Mesaverde (4912) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 684 4 476	9 232 2 814	0.097	0.051	0.091	0.051	2.500
Mesaverde (4946) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	13 449 4 099	7 696 2 346	0.077	-0.039	0.010	0.066	2.450
Mesaverde (5469.5) ² silty limestone	$P_{eff} = 27.58$ MPa saturated, undrained	16 312 4 972	9 512 2 899	0.056	-0.041	-0.003	0.067	2.630
Mesaverde (5481.3) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 270 4 349	8 434 2 571	0.091	0.134	0.148	0.105	2.460
Mesaverde (5501) ² clayshale	$P_{eff} = 27.58$ MPa saturated, undrained	12 887 3 928	6 742 2 055	0.334	0.818	0.730	0.575	2.590
Mesaverde (5555.5) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 891 4 539	8 877 2 706	0.060	0.147	0.143	0.045	2.480
Mesaverde (5566.3) ² laminated siltstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 596 4 449	8 482 2 585	0.091	0.688	0.565	0.046	2.570
Mesaverde (5837.5) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	15 327 4 672	9 294 2 833	0.023	-0.013	0.002	0.013	2.470
Mesaverde (5858.6) ² clayshale	$P_{eff} = 27.58$ MPa saturated, undrained	12 448 3 794	6 804 2 074	0.189	0.154	0.204	0.175	2.560
Mesaverde (6423.6) ² calcareous sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	17 914 5 460	10 560 3 219	0.000	-0.345	-0.264	-0.007	2.690
Mesaverde (6455.1) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 496 4 418	8 487 2 587	0.053	0.173	0.158	0.133	2.450
Mesaverde (6542.6) ² immature sandstone	$P_{eff} = 27.58$ MPa saturated, undrained	14 451 4 405	8 339 2 542	0.080	-0.057	-0.003	0.093	2.510

How to determine "F"

Anisotropic Parameters

$\alpha_0, \beta_0, \epsilon, \gamma, \delta$

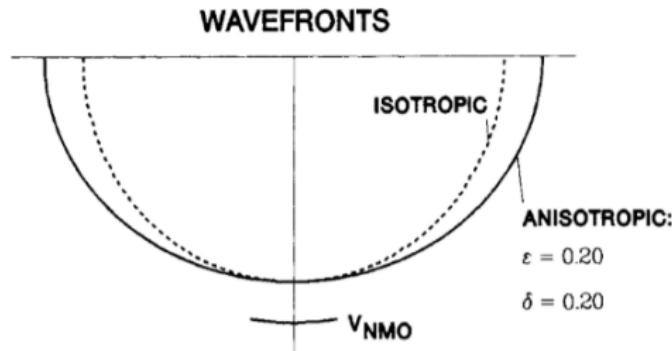


Figure 1: Example: P-wave propagation

How to determine "F"

Anisotropic Parameters

$\alpha_0, \beta_0, \epsilon, \gamma, \delta$

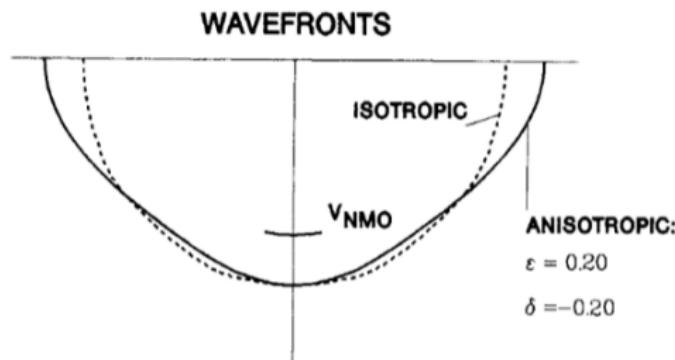


Figure 2: Example: P-wave propagation

How to determine "F"

Anisotropic Parameters

$\alpha_0, \beta_0, \epsilon, \gamma, \delta$

$$v_p(\theta) = \alpha_0(1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta),$$

$$v_{SV}(\theta) = \beta_0 \left[1 + \frac{\alpha_0^2}{\beta_0^2} (\epsilon - \delta) \sin^2 \theta \cos^2 \theta \right],$$

$$v_{SH}(\theta) = \beta_0(1 + \gamma \sin^2 \theta).$$

Thank you!