APPENDIX E

MATRICES FOR EQUILIBRIUM PROPERTIES IN THE 32 CRYSTAL CLASSES

TABLE 26 shows how the matrices that occur in Chapter X, equations (44) and (45), are affected by crystal symmetry in each of the 32 classes. For convenience of reference we repeat the equations here.

$$\left.\begin{aligned}
\epsilon_1 &= s_{11}^{E,T}\sigma_1+s_{12}\sigma_2+s_{13}\sigma_3+s_{14}\sigma_4+s_{15}\sigma_5+s_{16}\sigma_6 \;+\; d_{11}^{T}E_1+d_{21}E_2+d_{31}E_3 \;+\; \alpha_1^{E}\Delta T\\
\epsilon_2 &= s_{12}\sigma_1+s_{22}\sigma_2+s_{23}\sigma_3+s_{24}\sigma_4+s_{25}\sigma_5+s_{26}\sigma_6 \;+\; d_{12}E_1+d_{22}E_2+d_{32}E_3 \;+\; \alpha_2\Delta T\\
\epsilon_3 &= s_{13}\sigma_1+s_{23}\sigma_2+s_{33}\sigma_3+s_{34}\sigma_4+s_{35}\sigma_5+s_{36}\sigma_6 \;+\; d_{13}E_1+d_{23}E_2+d_{33}E_3 \;+\; \alpha_3\Delta T\\
\epsilon_4 &= s_{14}\sigma_1+s_{24}\sigma_2+s_{34}\sigma_3+s_{44}\sigma_4+s_{45}\sigma_5+s_{46}\sigma_6 \;+\; d_{14}E_1+d_{24}E_2+d_{34}E_3 \;+\; \alpha_4\Delta T\\
\epsilon_5 &= s_{15}\sigma_1+s_{25}\sigma_2+s_{35}\sigma_3+s_{45}\sigma_4+s_{55}\sigma_5+s_{56}\sigma_6 \;+\; d_{15}E_1+d_{25}E_2+d_{35}E_3 \;+\; \alpha_5\Delta T\\
\epsilon_6 &= s_{16}\sigma_1+s_{26}\sigma_2+s_{36}\sigma_3+s_{46}\sigma_4+s_{56}\sigma_5+s_{66}\sigma_6 \;+\; d_{16}E_1+d_{26}E_2+d_{36}E_3 \;+\; \alpha_6\Delta T\\
D_1 &= d_{11}^{T}\sigma_1+d_{12}\sigma_2+d_{13}\sigma_3+d_{14}\sigma_4+d_{15}\sigma_5+d_{16}\sigma_6 \;+\; \kappa_{11}^{\sigma,T}E_1+\kappa_{12}E_2+\kappa_{13}E_3 \;+\; p_1^{\sigma}\Delta T\\
D_2 &= d_{21}\sigma_1+d_{22}\sigma_2+d_{23}\sigma_3+d_{24}\sigma_4+d_{25}\sigma_5+d_{26}\sigma_6 \;+\; \kappa_{12}E_1+\kappa_{22}E_2+\kappa_{23}E_3 \;+\; p_2\Delta T\\
D_3 &= d_{31}\sigma_1+d_{32}\sigma_2+d_{33}\sigma_3+d_{34}\sigma_4+d_{35}\sigma_5+d_{36}\sigma_6 \;+\; \kappa_{13}E_1+\kappa_{23}E_2+\kappa_{33}E_3 \;+\; p_3\Delta T\\
\Delta S &= \alpha_1^{E}\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3+\alpha_4\sigma_4+\alpha_5\sigma_5+\alpha_6\sigma_6 \;+\; p_1^{\sigma}E_1+p_2E_2+p_3E_3 \;+\; (C^{\sigma,E}/T)\Delta T
\end{aligned}\right\};$$

Ch. X, (44)

$$\left.\begin{aligned}
\boldsymbol{\epsilon} &= \mathbf{s}^{E,T}\boldsymbol{\sigma}+\mathbf{d}_t^{T}\mathbf{E}+\boldsymbol{\alpha}^{E}\Delta T\\
\mathbf{D} &= \mathbf{d}^{T}\boldsymbol{\sigma}+\boldsymbol{\kappa}^{\sigma,T}\mathbf{E}+\mathbf{p}^{\sigma}\Delta T\\
\Delta S &= \boldsymbol{\alpha}_t^{E}\boldsymbol{\sigma}+\mathbf{p}_t^{\sigma}\mathbf{E}+(C^{\sigma,E}/T)\Delta T
\end{aligned}\right\}.$$

Ch. X, (45)

Table 26 collects together the matrices given in the text on pp. 23, 79, 123–4, 140–1. All the coefficients on the right-hand side of the above equations are presented as one matrix $[(6+3+1)\times(6+3+1)] = [10\times 10]$ thus:

	σ	E	ΔT
ϵ	**s**	$\mathbf{d}_t$	$\boldsymbol{\alpha}$
D	**d**	$\boldsymbol{\kappa}$	**p**
ΔS	$\boldsymbol{\alpha}_t$	$\mathbf{p}_t$	C/T

s = elastic compliances
d = piezoelectric moduli
α = thermal expansion coefficients
κ = permittivities
p = pyroelectric coefficients
C = heat capacity
T = absolute temperature

At the side of each matrix is given the number of independent coefficients for each property in the order, **s**, **d**, **α**, **κ**, **p**, C/T, with the total number at the bottom. The setting of the reference axes x_1, x_2, x_3, relative to the symmetry elements follows the conventions given in Appendix B. All the 10×10 matrices are symmetrical about the leading diagonal. All other equalities and relations between the elements demanded by the point-group symmetry are shown by the symbolism explained at the head of the table; this is the same as the symbolism used in the main text.

TABLE 26

Matrices for equilibrium properties in the 32 crystal classes

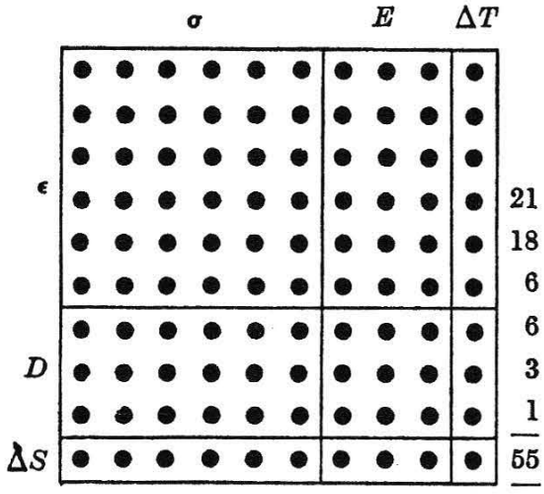
KEY TO NOTATION

• zero component ● non-zero component ●—● equal components
●—○ components numerically equal, but opposite in sign
◉ a component equal to twice the heavy dot component to which it is joined
◎ a component equal to minus 2 times the heavy dot component to which it is joined
✕ $2(s_{11}-s_{12})$

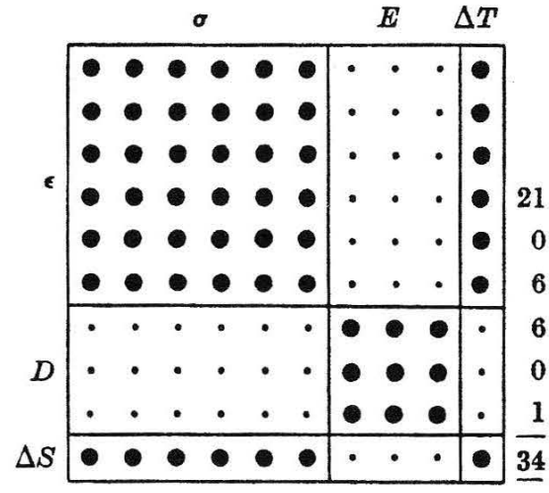
Each complete 10×10 matrix is symmetrical about the leading diagonal.

TRICLINIC SYSTEM

Class *1*

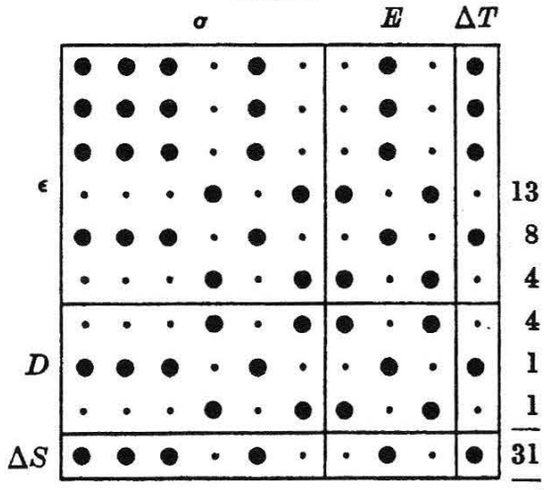


Class $\bar{1}$

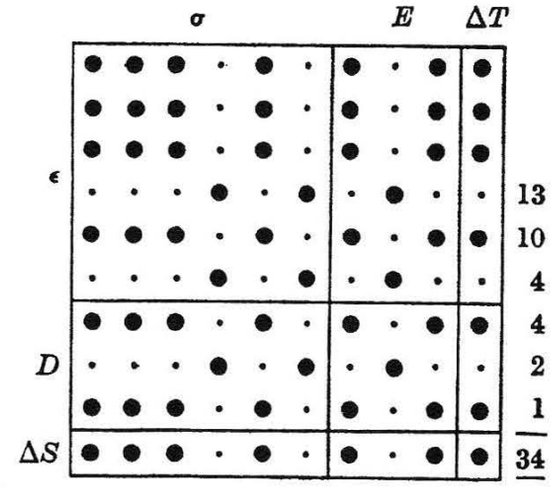


MONOCLINIC SYSTEM

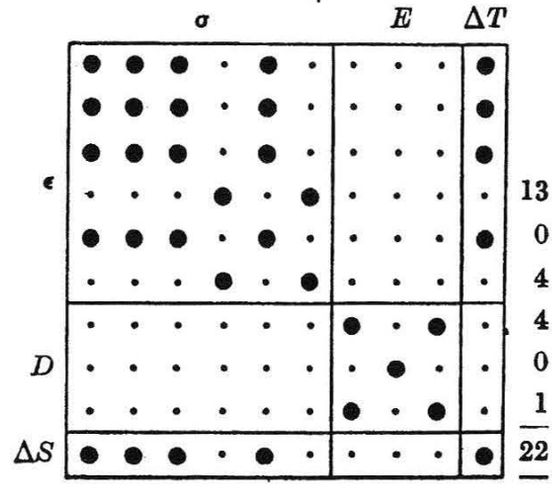
Class *2*



Class *m*

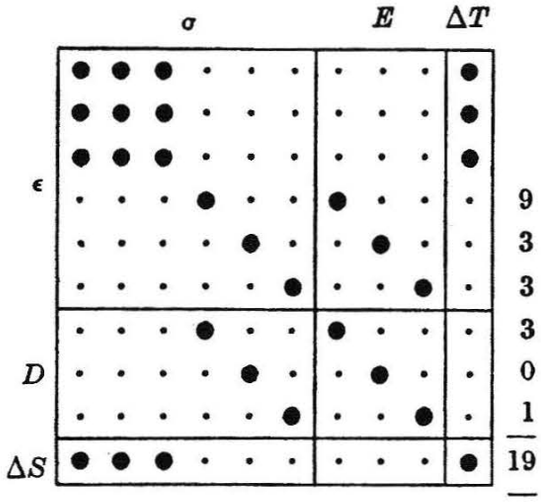


Class *2/m*

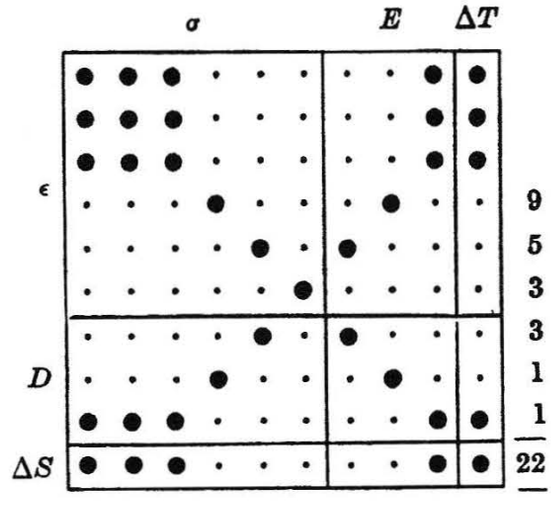


Orthorhombic System

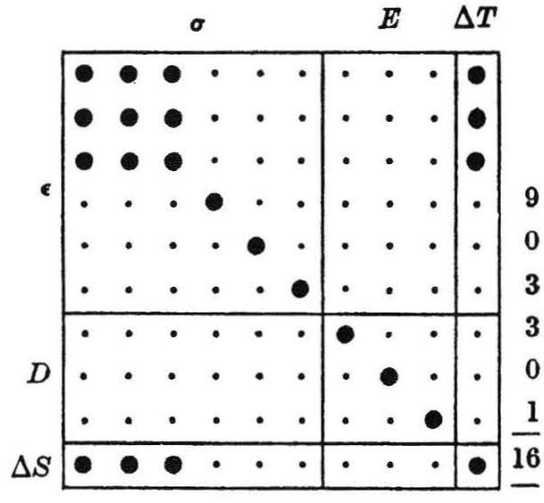
Class *222*



Class *mm2*

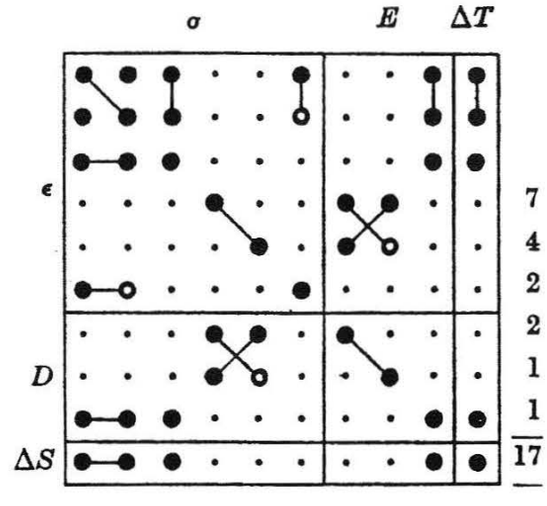


Class *mmm*

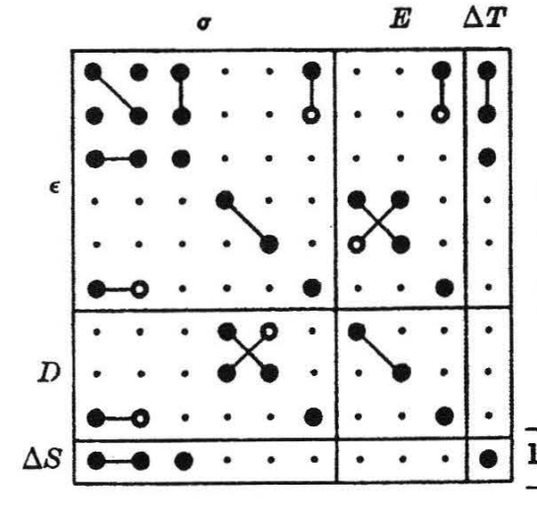


Tetragonal System

Class *4*

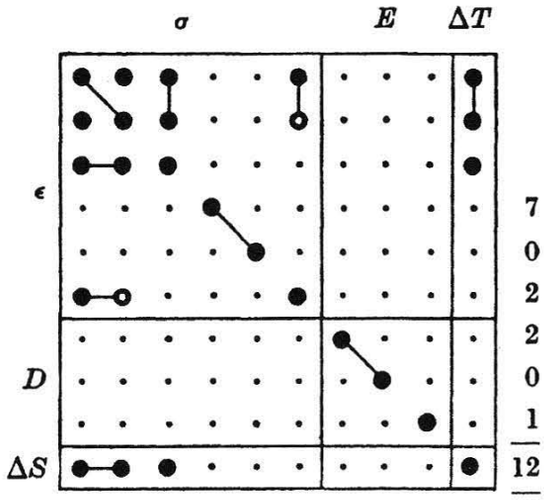


Class $\bar{4}$

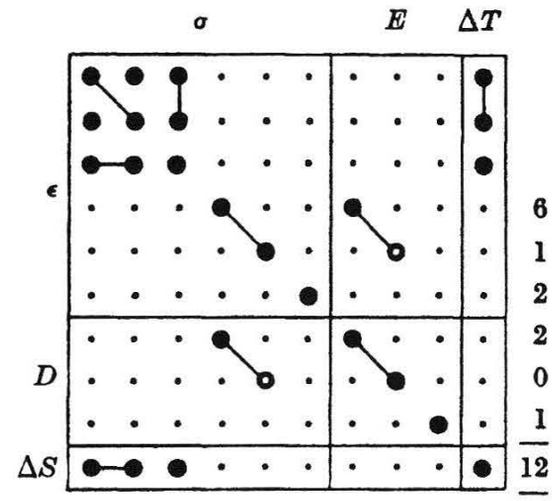


Tetragonal System (*continued*)

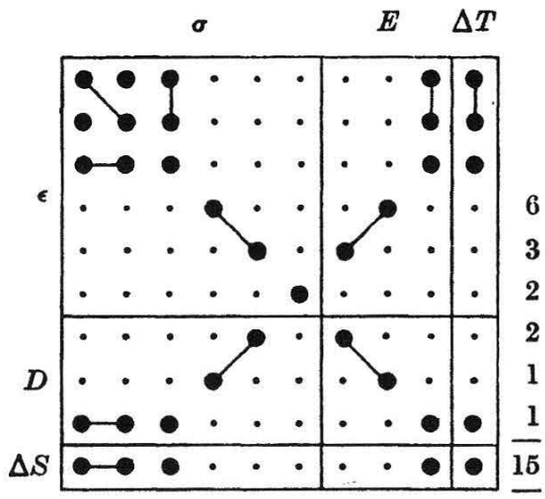
Class *4/m*



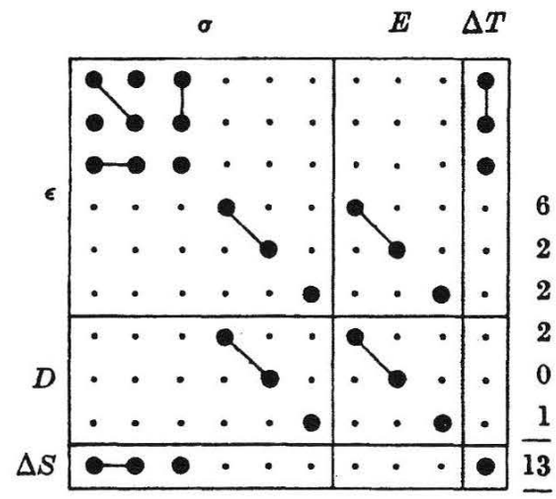
Class *422*



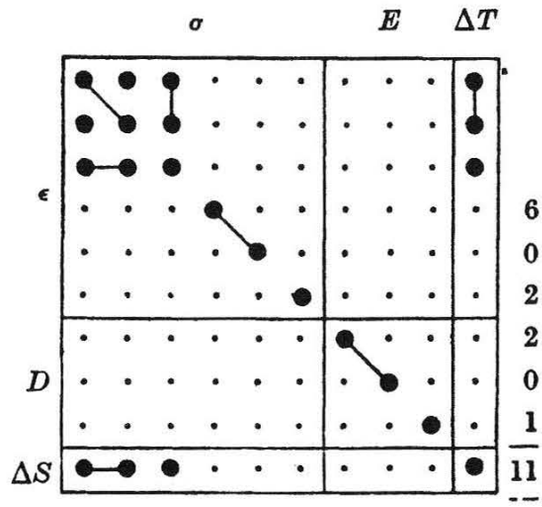
Class *4mm*



Class $\bar{4}2m$

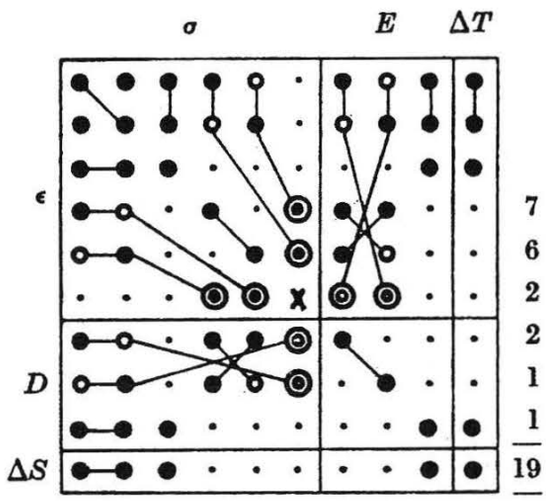


Class *4/mmm*

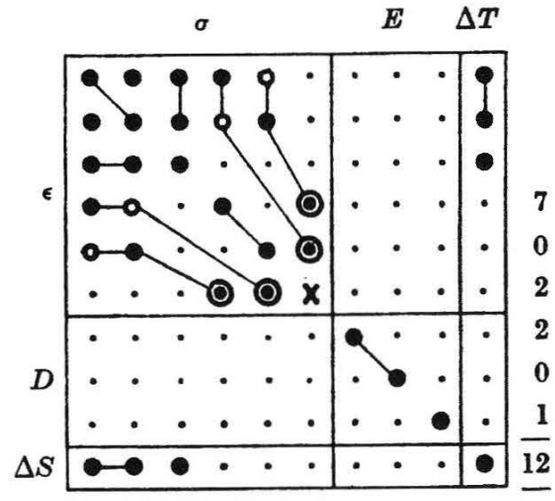


Trigonal System

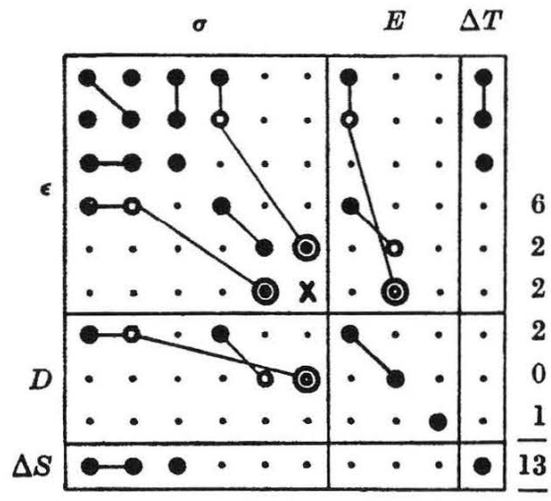
Class 3



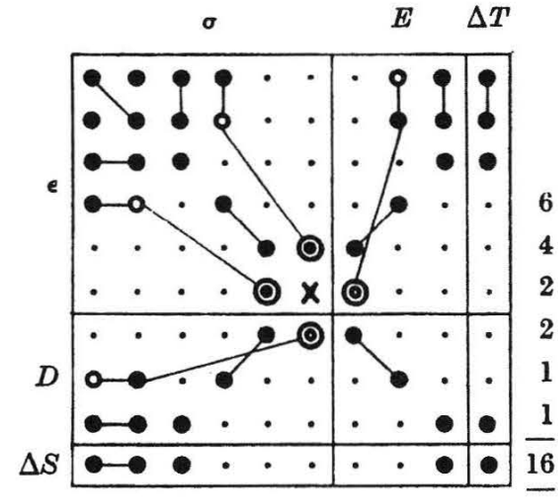
Class $\bar{3}$



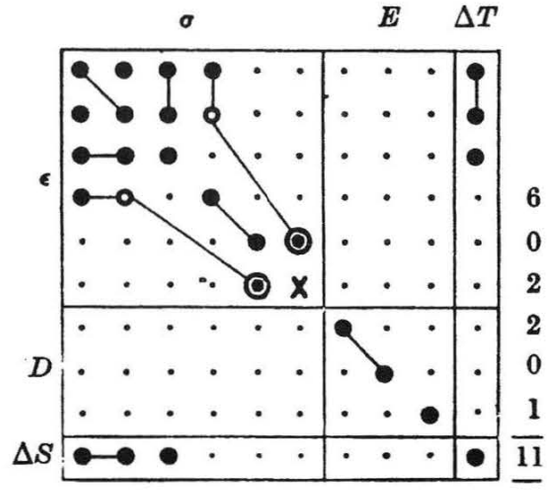
Class 32



Class 3m

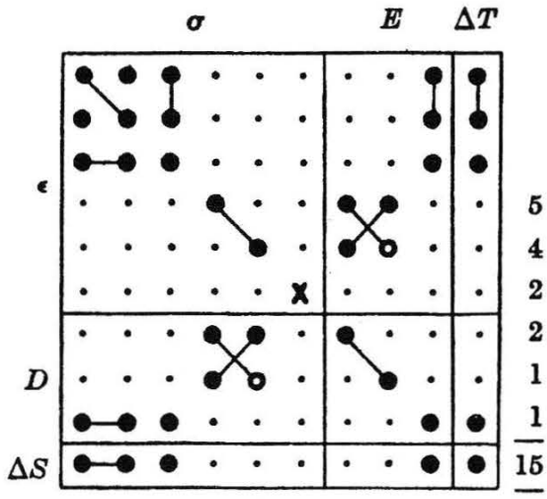


Class $\bar{3}m$

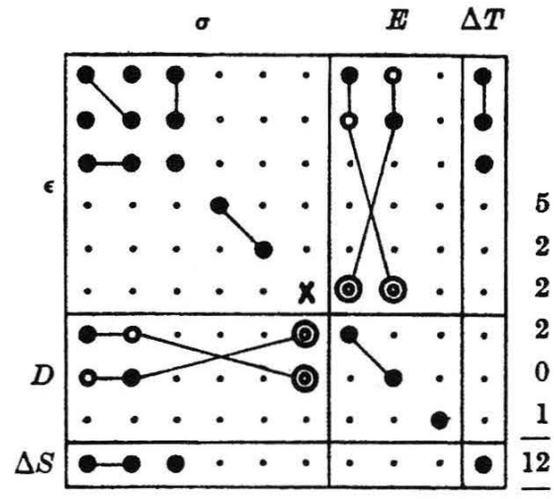


Hexagonal System

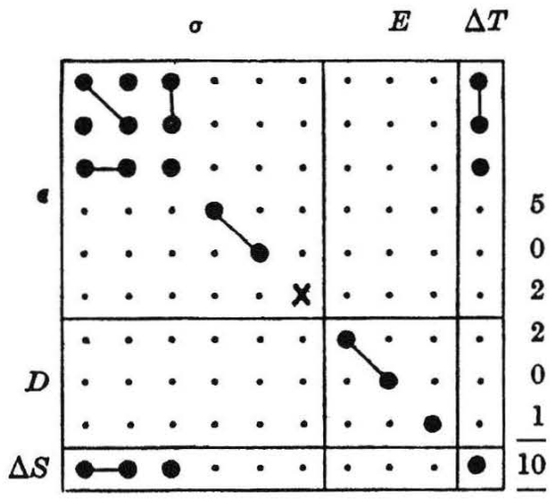
Class 6



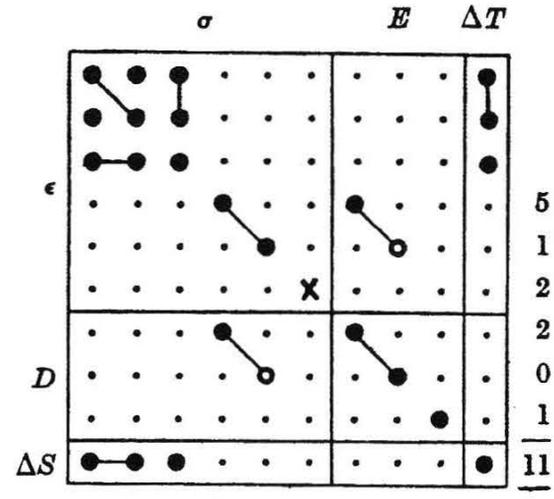
Class $\bar{6}$



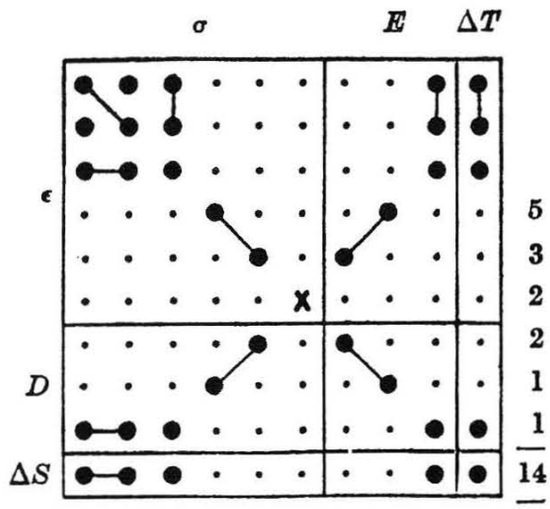
Classes 6/m and 6/mmm



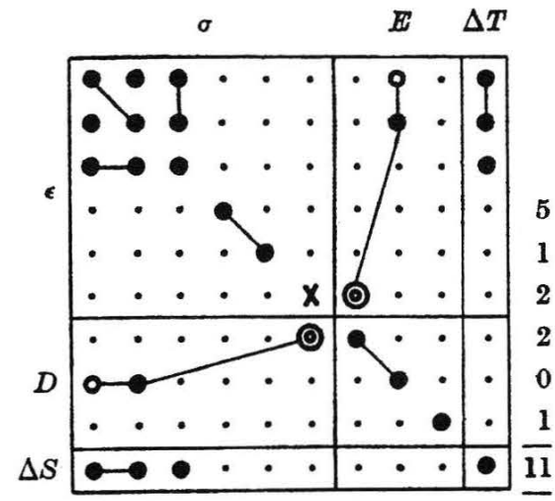
Class 622



Class 6mm

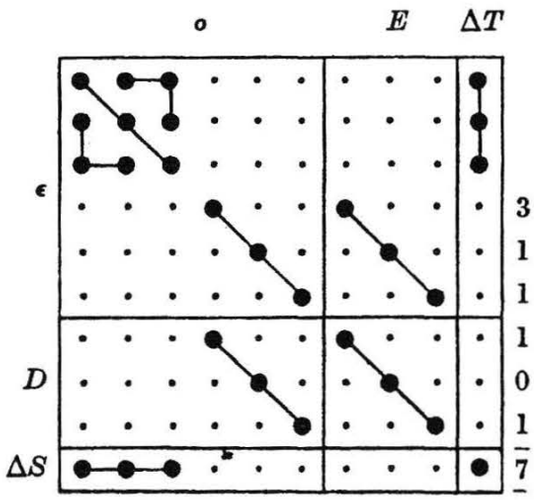


Class $\bar{6}m2$

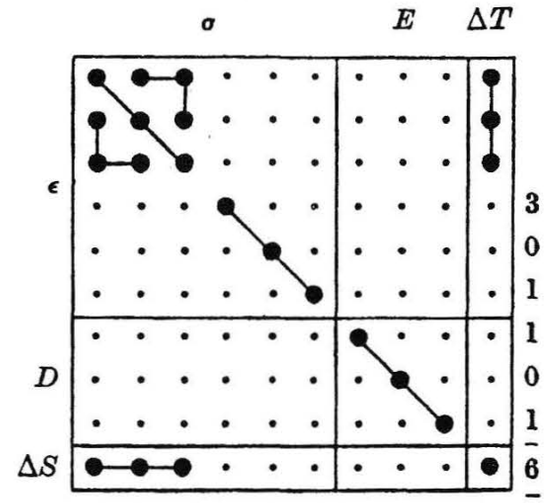


Cubic System

Classes *23* and $\bar{4}3m$



Classes *m3*, *432* and *m3m*



Isotropic