

Source Encoding for Adjoint Tomography

Tromp et al. 2019

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Source Encoding

Randomly assign frequencies to each source $s = 1, \dots, S$:

$$\omega_s = \omega_{\min} + (s - 1) \Delta\omega,$$

where

$$\Delta\omega = (\omega_{\max} - \omega_{\min})/(S - 1),$$

Source Encoding

Use orthogonality of the steady-state wavefield:

$$\frac{1}{\Delta\tau} \int_{T_{ss}}^{T_{ss} + \Delta\tau} \exp(-i\omega_s t) \exp(i\omega_{s'} t) dt = \delta_{ss'},$$

where

$$\Delta\tau = \frac{2\pi}{\Delta\omega} = \frac{2\pi(S - 1)}{\omega_{\max} - \omega_{\min}}.$$

Source Encoding

Super forward source :

$$F_j(\mathbf{x}, t) = \Re \sum_{s=1}^S f_j^s(\mathbf{x}, \omega_s) \exp(i\omega_s t),$$

Super forward wavefield:

$$S_i(\mathbf{x}, t) = \int_{-\infty}^t \int G_{ij}(\mathbf{x}, \mathbf{x}'; t - t') F_j(\mathbf{x}', t') d^3\mathbf{x}' dt'.$$

Source Encoding

Steady state forward wavefield:

$$S_i(\mathbf{x}, t) = \sum_{s=1}^S [A_i^s(\mathbf{x}) \cos(\omega_s t) + B_i^s(\mathbf{x}) \sin(\omega_s t)].$$

We can “decode” or “deblend” its stationary parts using orthogonality:

$$A_i^s(\mathbf{x}) = \frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss}+\Delta\tau} S_i(\mathbf{x}, t) \cos(\omega_s t) dt,$$

$$B_i^s(\mathbf{x}) = \frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss}+\Delta\tau} S_i(\mathbf{x}, t) \sin(\omega_s t) dt.$$

Source Encoding

Encoded misfit function:

$$\chi = \frac{1}{2} \sum_{s=1}^S \sum_{r=1}^{R_s} [s_i^{s*}(\mathbf{x}_r) - d_i^{s*}(\mathbf{x}_r)][s_i^s(\mathbf{x}_r) - d_i^s(\mathbf{x}_r)].$$

Source Encoding

Super adjoint source :

$$F_j^\dagger(\mathbf{x}, t) = \Re \sum_{s=1}^S f_j^{\dagger s}(\mathbf{x}, \omega_s) \exp(i\omega_s t),$$

Super adjoint wavefield:

$$S_i^\dagger(\mathbf{x}, t) = \int_{-\infty}^t \int G_{ij}(\mathbf{x}, \mathbf{x}'; t - t') F_j^{\dagger s}(\mathbf{x}', t') d^3\mathbf{x}' dt'.$$

Source Encoding

Steady state adjoint wavefield:

$$S_i^\dagger(\mathbf{x}, t) = \sum_{s=1}^S [A_i^{\dagger s}(\mathbf{x}) \cos(\omega_s t) - B_i^{\dagger s}(\mathbf{x}) \sin(\omega_s t)]$$

We can “decode” or “deblend” its stationary parts using orthogonality:

$$A_i^{\dagger s}(\mathbf{x}) = \frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss} + \Delta\tau} S_i^\dagger(\mathbf{x}, t) \cos(\omega_s t) dt,$$

$$B_i^{\dagger s}(\mathbf{x}) = \frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss} + \Delta\tau} S_i^\dagger(\mathbf{x}, t) \sin(\omega_s t) dt.$$

Source Encoding

Fréchet derivative:

$$\delta\chi = \int (\delta \ln \rho K_\rho + \delta \ln \kappa K_\kappa + \delta \ln \mu K_\mu) d^3\mathbf{x},$$

where

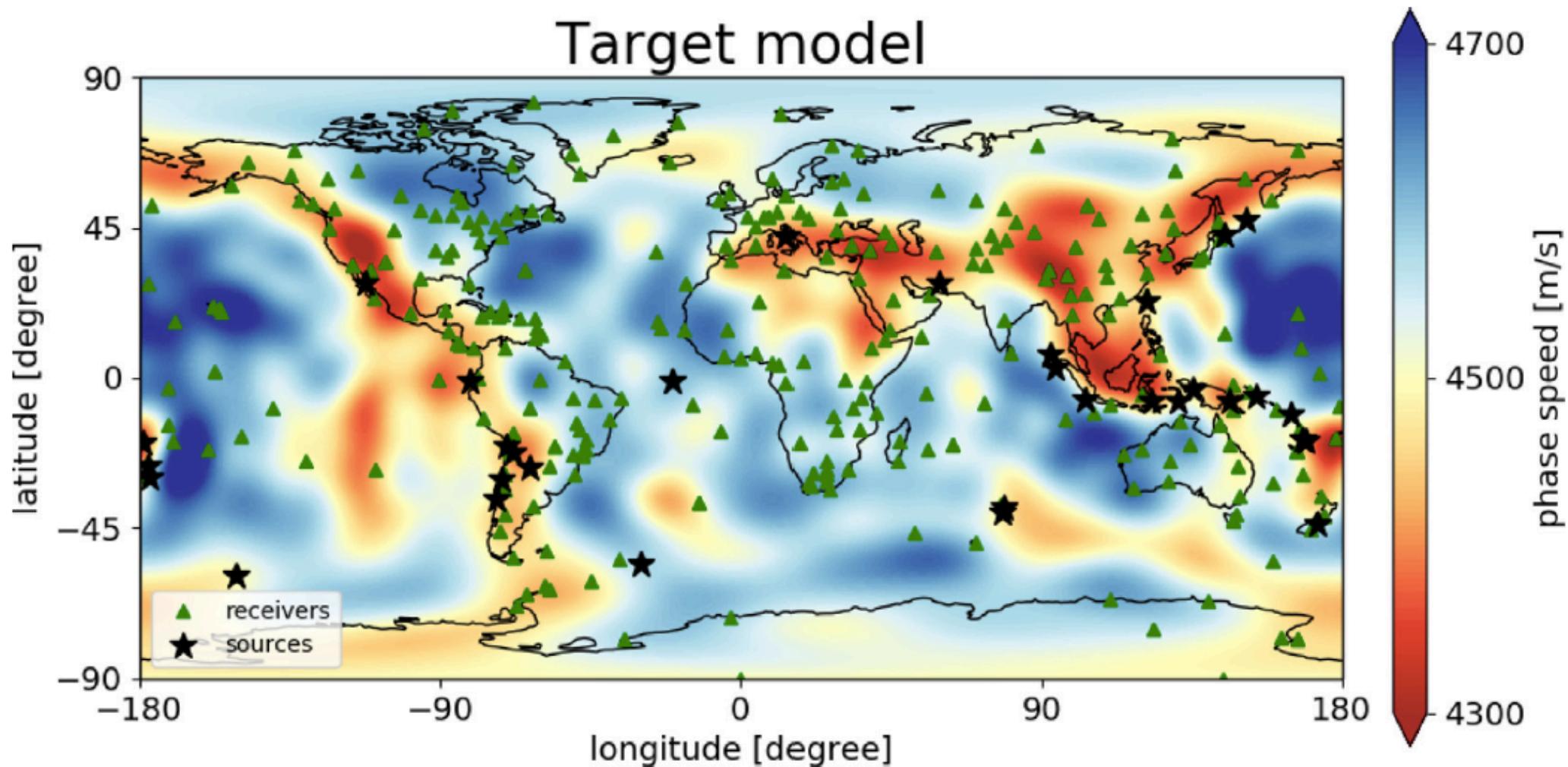
$$\begin{aligned} K_\rho(\mathbf{x}) &= -\frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss}+\Delta\tau} \rho(\mathbf{x}) S_i^\dagger(\mathbf{x}, -t) \partial_t^2 S_i(\mathbf{x}, t) dt \\ &= \sum_{s=1}^S \omega_s^2 \rho(\mathbf{x}) [A_i^{\dagger s}(\mathbf{x}) A_i^s(\mathbf{x}) + B_i^{\dagger s}(\mathbf{x}) B_i^s(\mathbf{x})] \end{aligned}$$

$$\begin{aligned}
 K_\kappa(\mathbf{x}) &= -\frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss}+\Delta\tau} \kappa(\mathbf{x}) [\nabla_i S_i^\dagger(\mathbf{x}, -t)] [\nabla_j S_j(\mathbf{x}, t)] dt \\
 &= -\sum_{s=1}^S \kappa(\mathbf{x}) \{ [\nabla_i A_i^{\dagger s}(\mathbf{x})] [\nabla_j A_j^s(\mathbf{x})] + [\nabla_i B_i^{\dagger s}(\mathbf{x})] [\nabla_j B_j^s(\mathbf{x})] \} \\
 &= -\Re \sum_{s=1}^S \kappa(\mathbf{x}) [\nabla_i s_i^{\dagger s*}(\mathbf{x})] [\nabla_j s_j^s(\mathbf{x})],
 \end{aligned}$$

$$\begin{aligned}
 K_\mu(\mathbf{x}) &= -\frac{2}{\Delta\tau} \int_{T_{ss}}^{T_{ss}+\Delta\tau} 2\mu(\mathbf{x}) D_{ij}^\dagger(\mathbf{x}, -t) D_{ij}(\mathbf{x}, t) dt \\
 &= -\sum_{s=1}^S 2\mu(\mathbf{x}) \{ [\tfrac{1}{2}(\nabla_i A_j^{\dagger s} + \nabla_j A_i^{\dagger s}) - \tfrac{1}{3} \nabla_k A_k^{\dagger s} \delta_{ij}] [\tfrac{1}{2}(\nabla_i A_j^s + \nabla_j A_i^s) - \tfrac{1}{3} \nabla_k A_k^s \delta_{ij}] \\
 &\quad + [\tfrac{1}{2}(\nabla_i B_j^{\dagger s} + \nabla_j B_i^{\dagger s}) - \tfrac{1}{3} \nabla_k B_k^{\dagger s} \delta_{ij}] [\tfrac{1}{2}(\nabla_i B_j^s + \nabla_j B_i^s) - \tfrac{1}{3} \nabla_k B_k^s \delta_{ij}] \} \\
 &= -\Re \sum_{s=1}^S 2\mu(\mathbf{x}) D_{ij}^{\dagger s*}(\mathbf{x}) D_{ij}^s(\mathbf{x}),
 \end{aligned}$$

Source Encoding

293 stations, 32 sources, 50 s Rayleigh waves

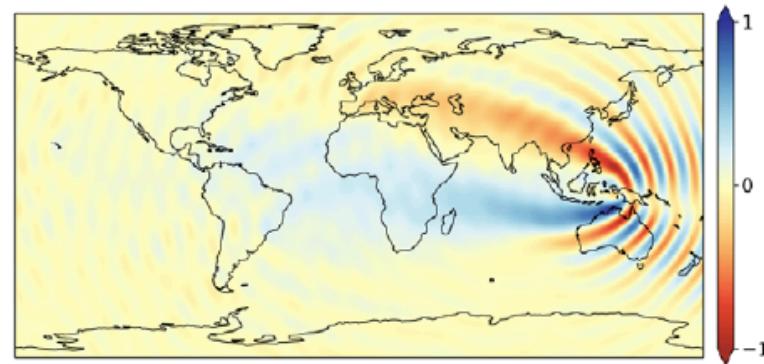


Source Encoding

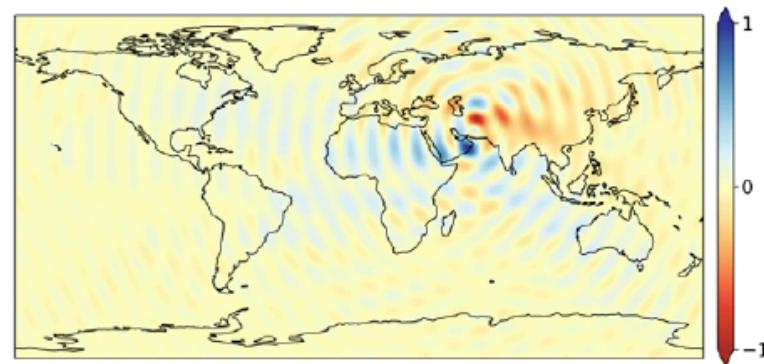
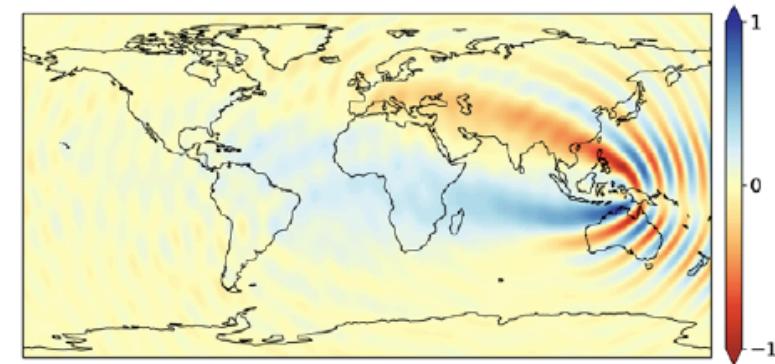
Event 1
(200.0 Hz)

Event 2
(200.012 Hz)

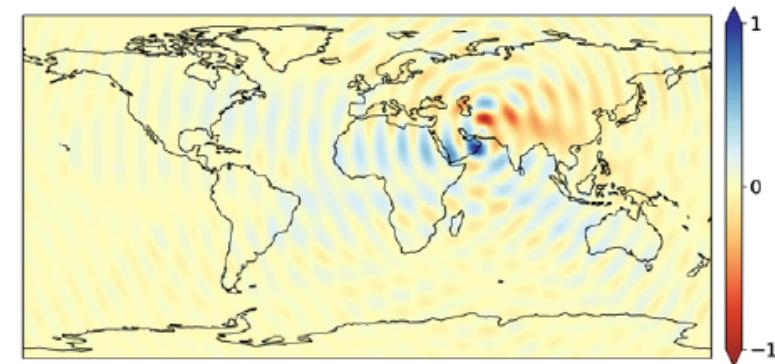
Individual simulation



Decoded super simulation



64 simulations

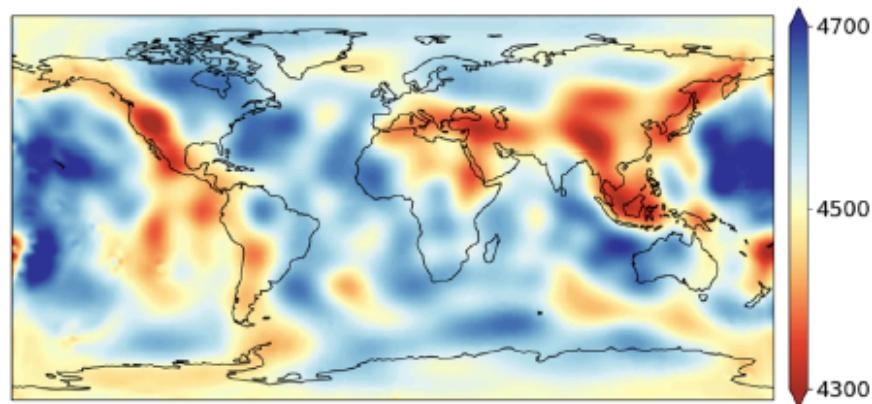


2 simulations

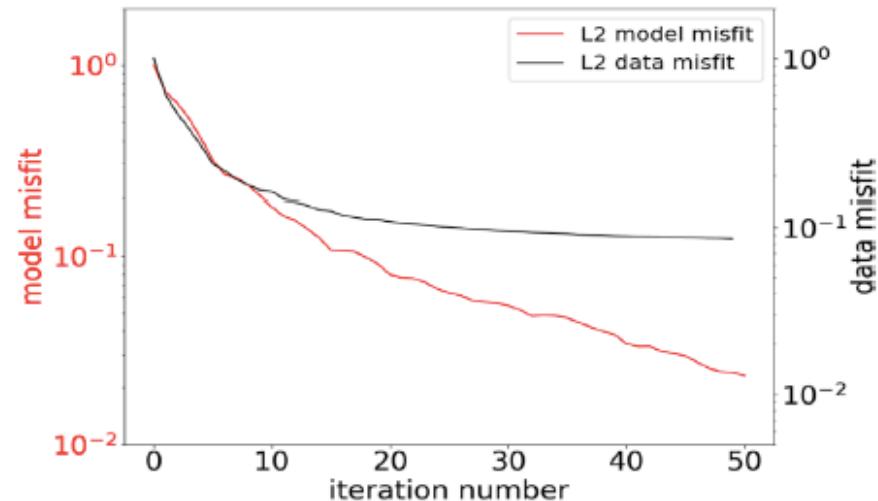
512 frequencies of each of the 32 sources: total of 16,384 frequencies

Source Encoding

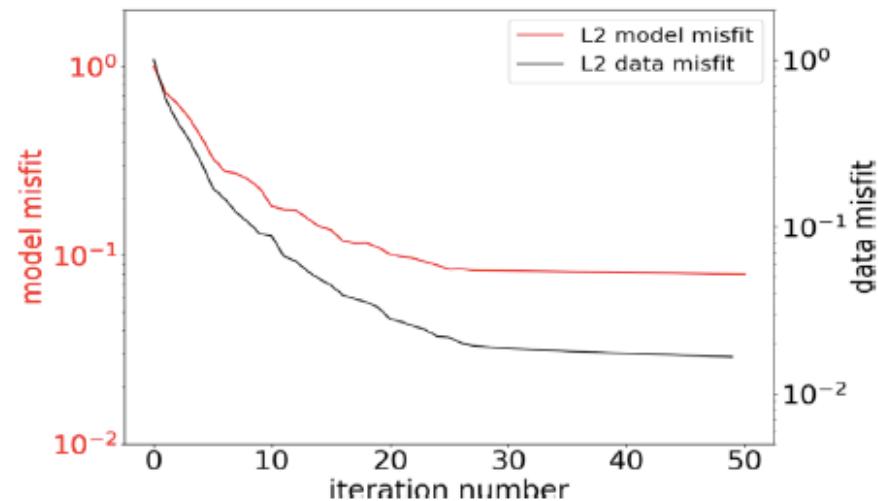
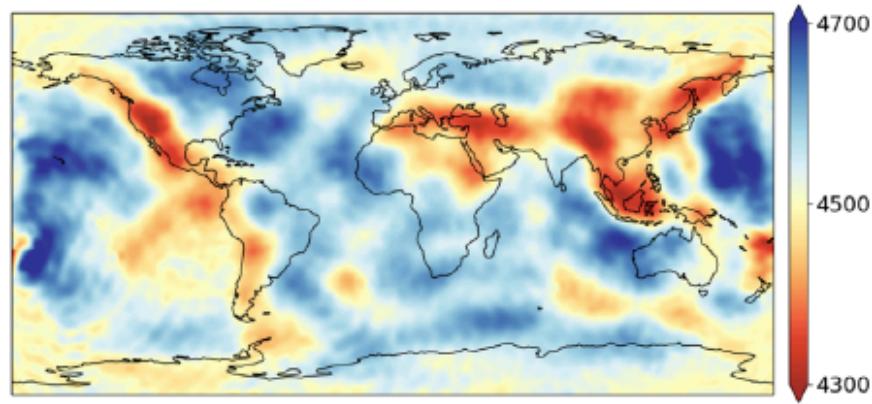
Traditional



L2 differences



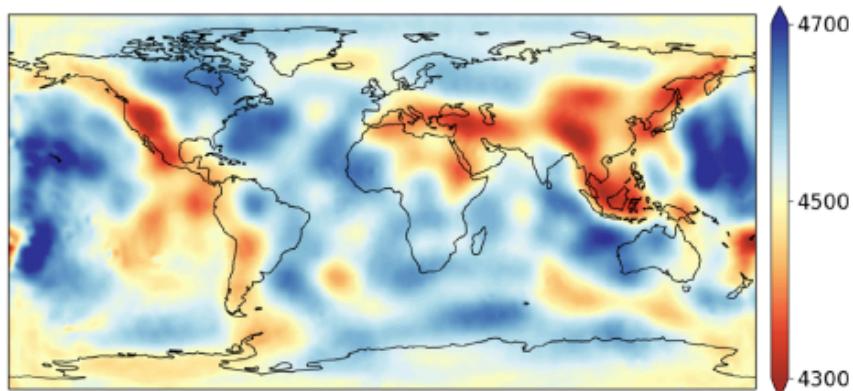
Source encoded



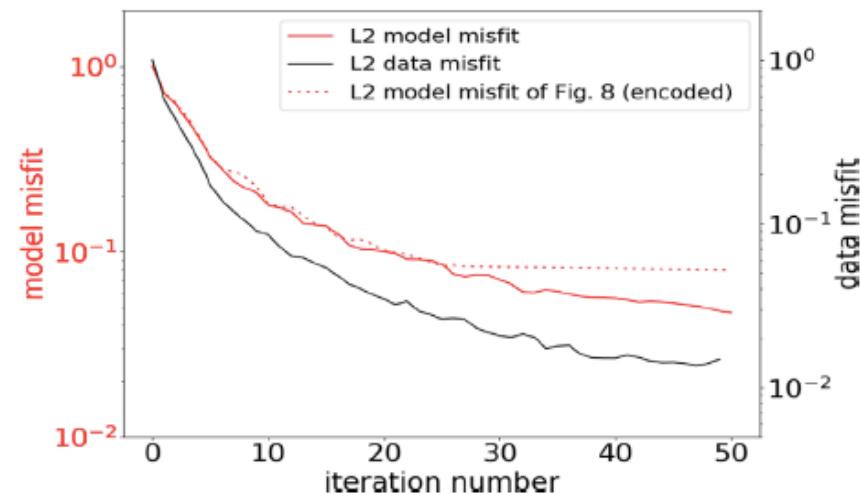
Source Encoding

Randomized
frequencies

Iteration 50



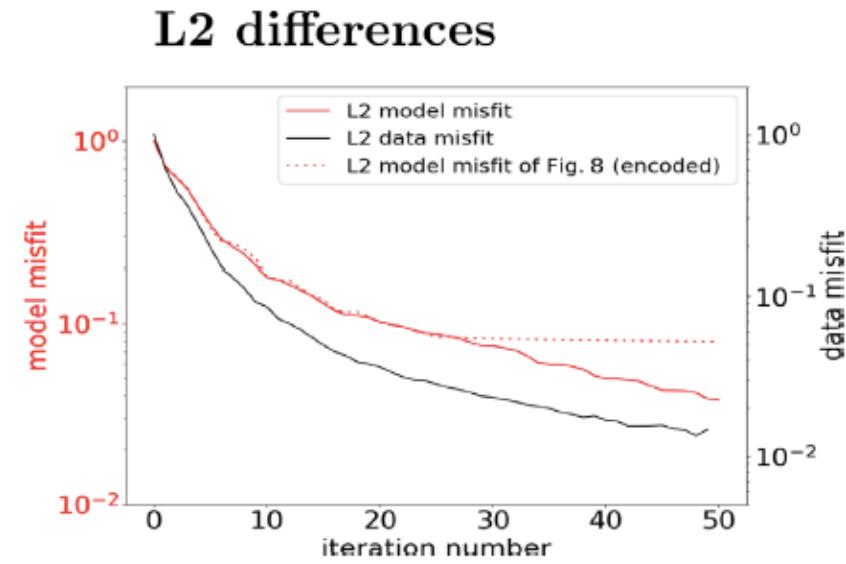
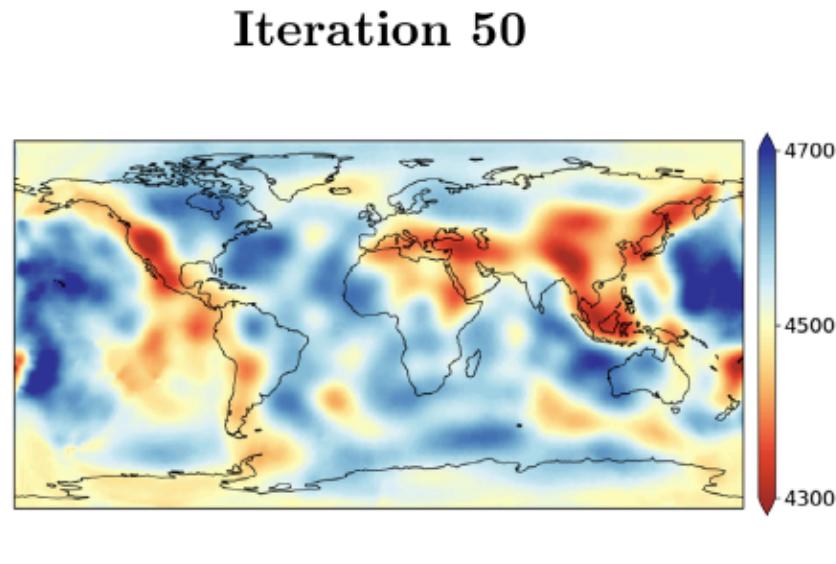
L2 differences



Instead of assigning the same frequency for an event for entire interation,
the frequency is changes at each iteration, resulting in increased
convergence rate

Source Encoding

3 frequencies
per event

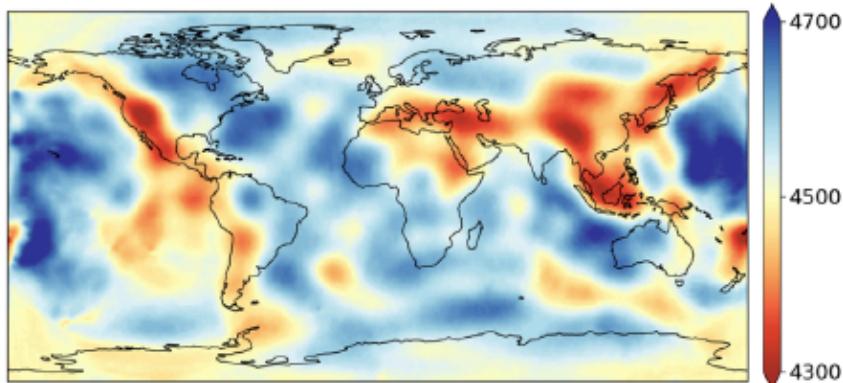


Faster convergence in model space!

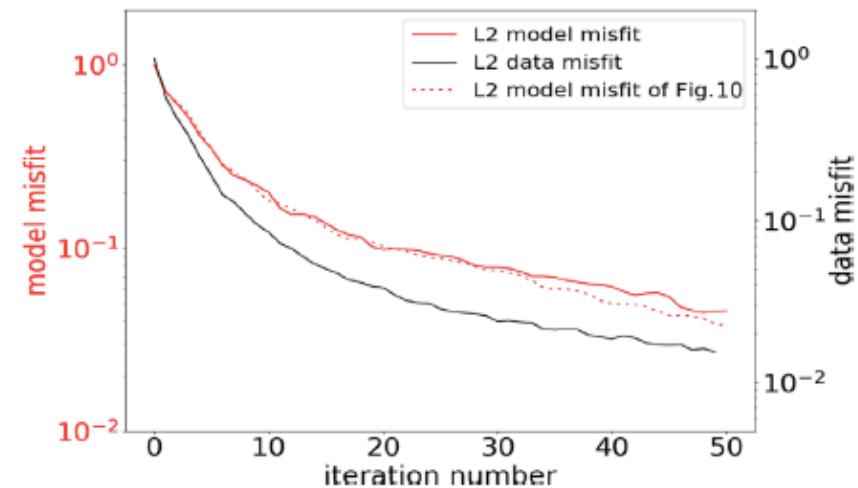
Source Encoding

30% of operating
stations per event

Iteration 50



L2 differences

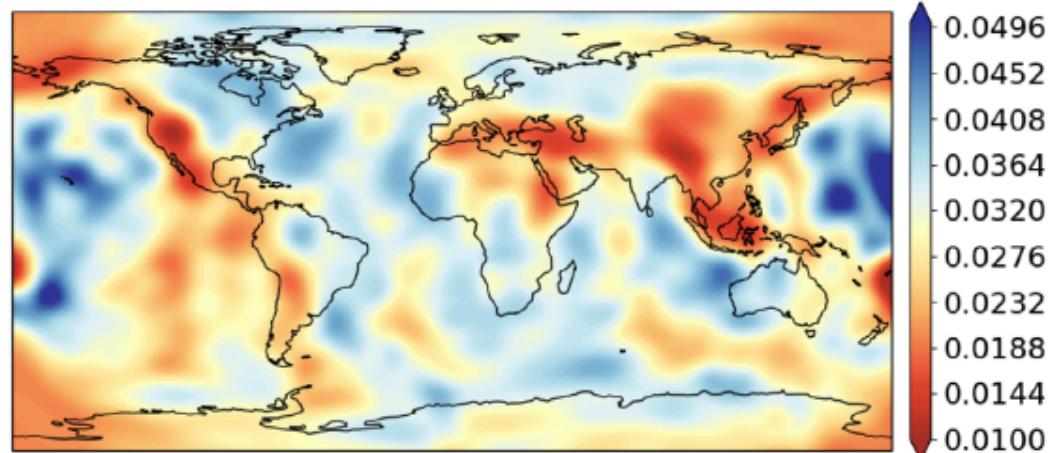


More tolerate for noise

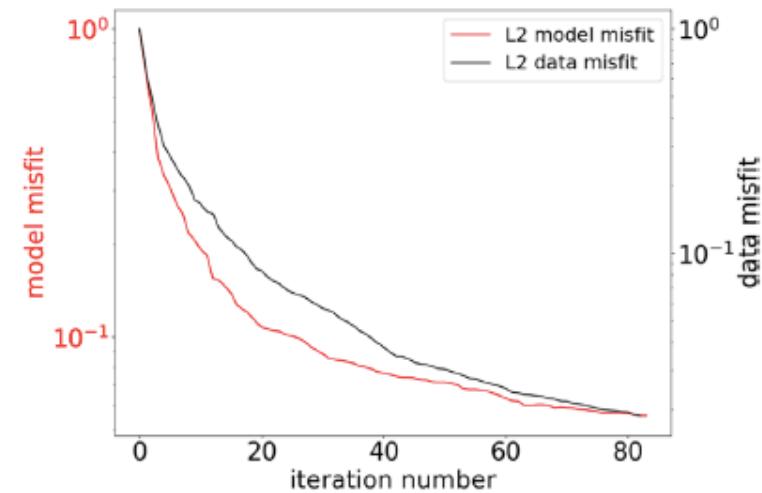
Source Encoding

Joint wave speed & Q inversion

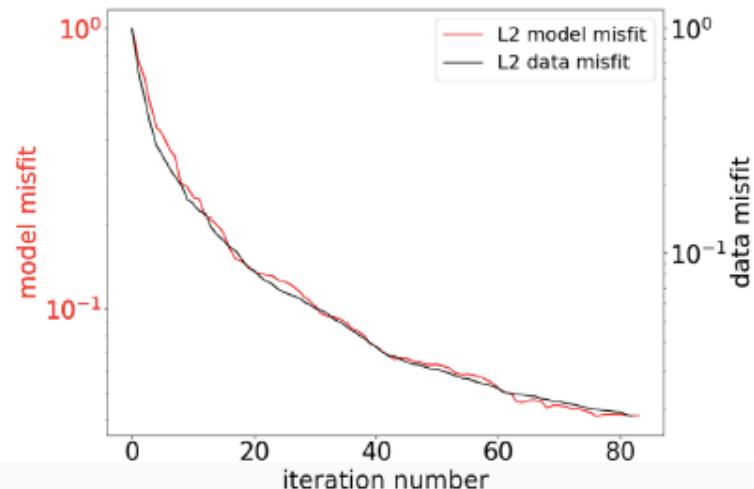
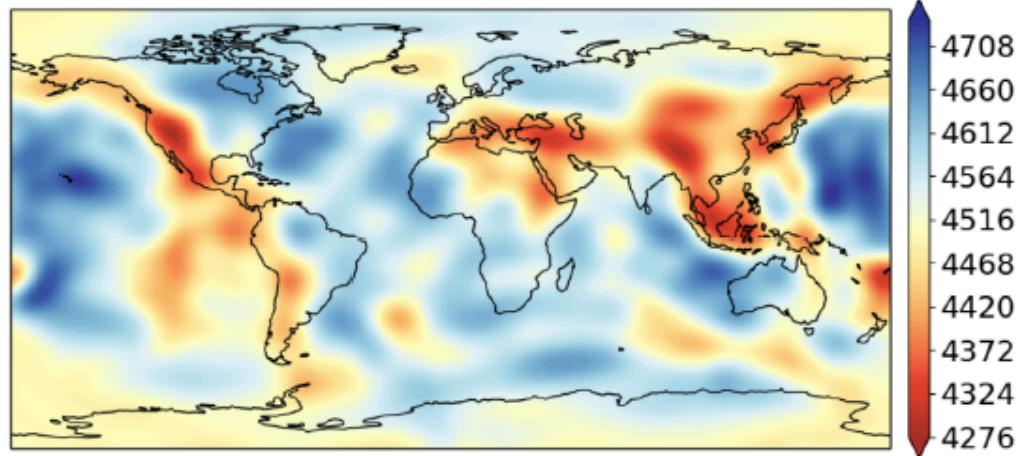
Iteration 80



L2-difference



300 Hz phase speed



Computational cost analysis

$$T_{traditional} = 3TS$$

$$T_{encoded+measurements} = \begin{cases} TS & measurements \\ 2(T_{ss} + \Delta\tau) & super forward and adjoint \\ \Delta\tau & steady super forward reconstruction \end{cases}$$

$$T_{encoded} = \begin{cases} 2(T_{ss} + \Delta\tau) & super forward and adjoint \\ \Delta\tau & steady super forward reconstruction \end{cases}$$

$$\frac{T_{traditional}}{T_{encoded+measurements}} \approx \frac{1}{\frac{1}{3} + \frac{1}{T(f_{max}-f_{min})}}$$

$$\frac{T_{traditional}}{T_{encoded}} \approx T(f_{max} - f_{min})$$

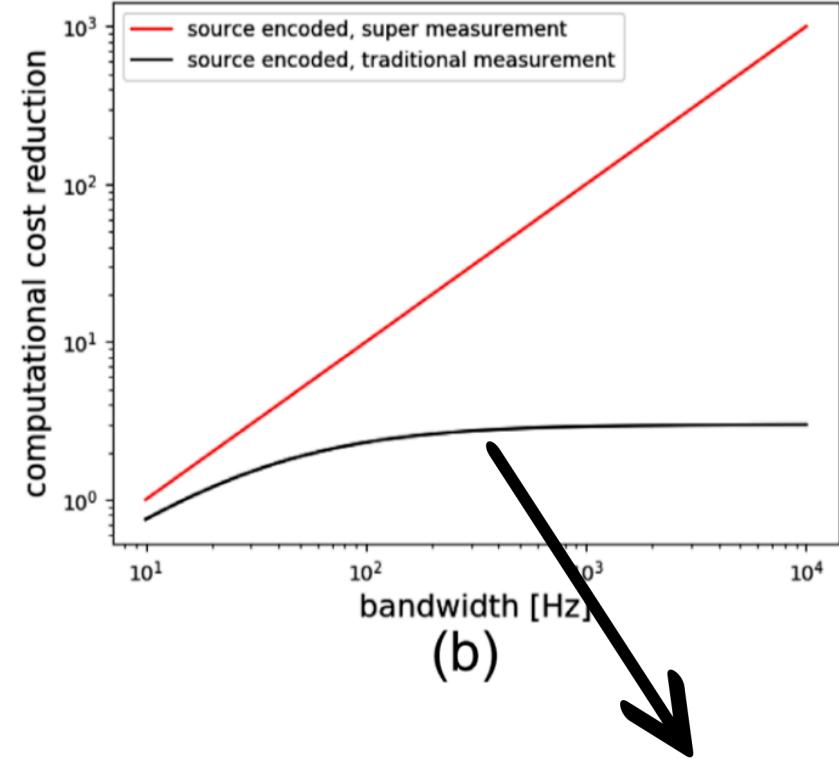
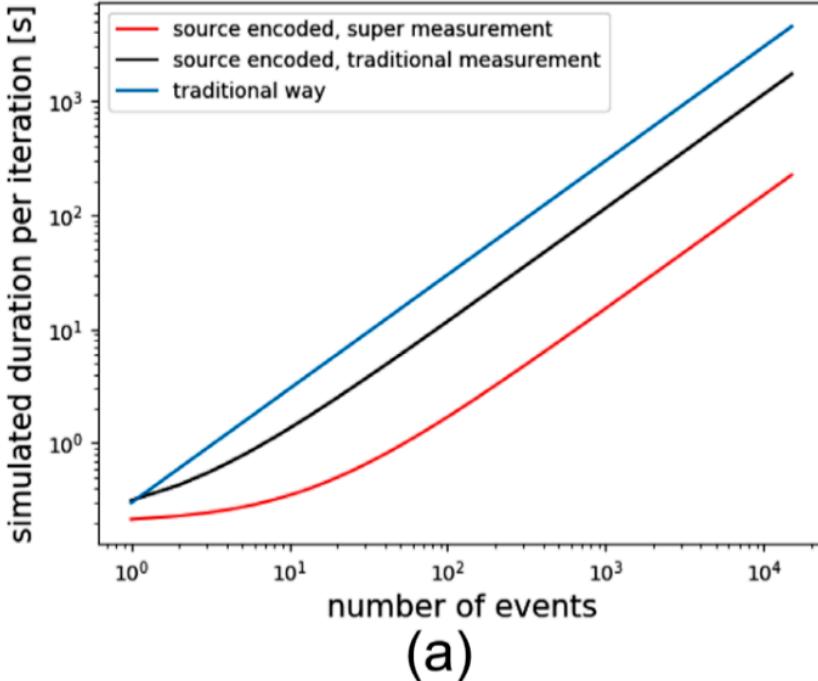
Storage analysis

$$I/O_{traditional} = \alpha T_{traditional} = \alpha 3TS$$

$$I/O_{encoded} = \alpha \frac{3(S-1)}{f_{max} - f_{min}}$$

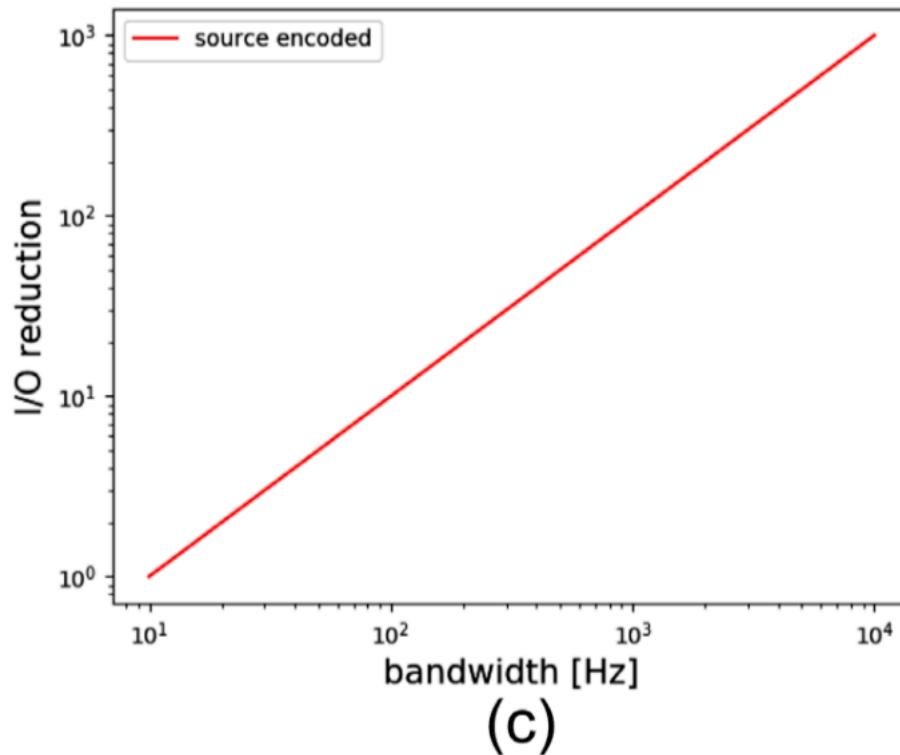
$$I/O_{encoded} \approx I/O_{encoded+measurements}$$

$$\frac{I/O_{traditional}}{I/O_{encoded+measurements}} \approx T(f_{max} - f_{min})$$



Bandwidth of 0.05 Hz: 360X reduction
 Bandwidth of 1 Hz: 7200X reduction

Not more than 3X



Exascale Seismology Science Goals

- Use data with a shortest period of ~ 1 Hz
- Use all available events with magnitudes greater than ~ 5.5
- Use entire 200 minutes long, three-component seismograms
- Workflow stabilization & management
- Allow for transverse isotropy with a random symmetry axis
- Allow for variations in attenuation
- Facilitate uncertainty quantification
- Source encoding to reduce the cost of the gradient calculation
- Explore possibility of Hamiltonian Monte Carlo with source encoding
- Opportunities for ML / AI in data selection & assimilation
- Data mining, feature extraction & visualization