AY2019/2020 Semester 1 Conference (Week 6)

Grid Search for the Earthquake (Re)location

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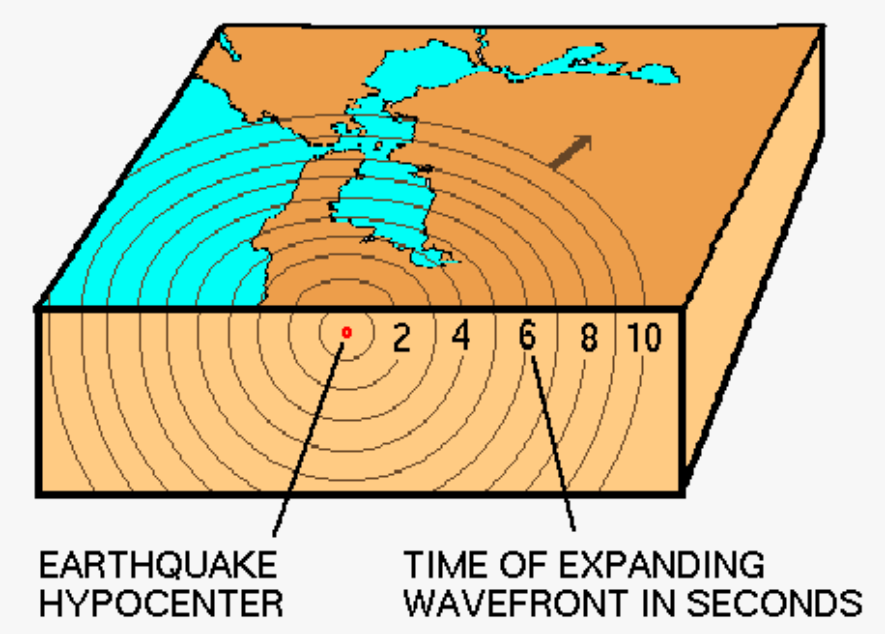
Earthquake Location

Hypocenter/Earthquake Focus: (x_0, y_0, z_0), Origin Time: t_0



For the hypocenter, (x_0, y_0) is in latitude and longitude coordinates where the earthquake takes place, while z_0 is the depth of the hypocenter in kilometres (km). While, for the origin time, t_0 is the time which the earthquake takes place in seconds (s).

Re-call: Typical Criteria for Manual Phase Picking

Primary Wave (P Wave/Phase): This wave has an estimated velocity of around $v = 6\ km/s$ and arrives first as it travels directly from the Earthquake focus point to the Seismic Station. It is a longitudinal wave, that is a wave that travels in a direction parallel to the direction of vibration, which creates regions of compression and rarefaction.

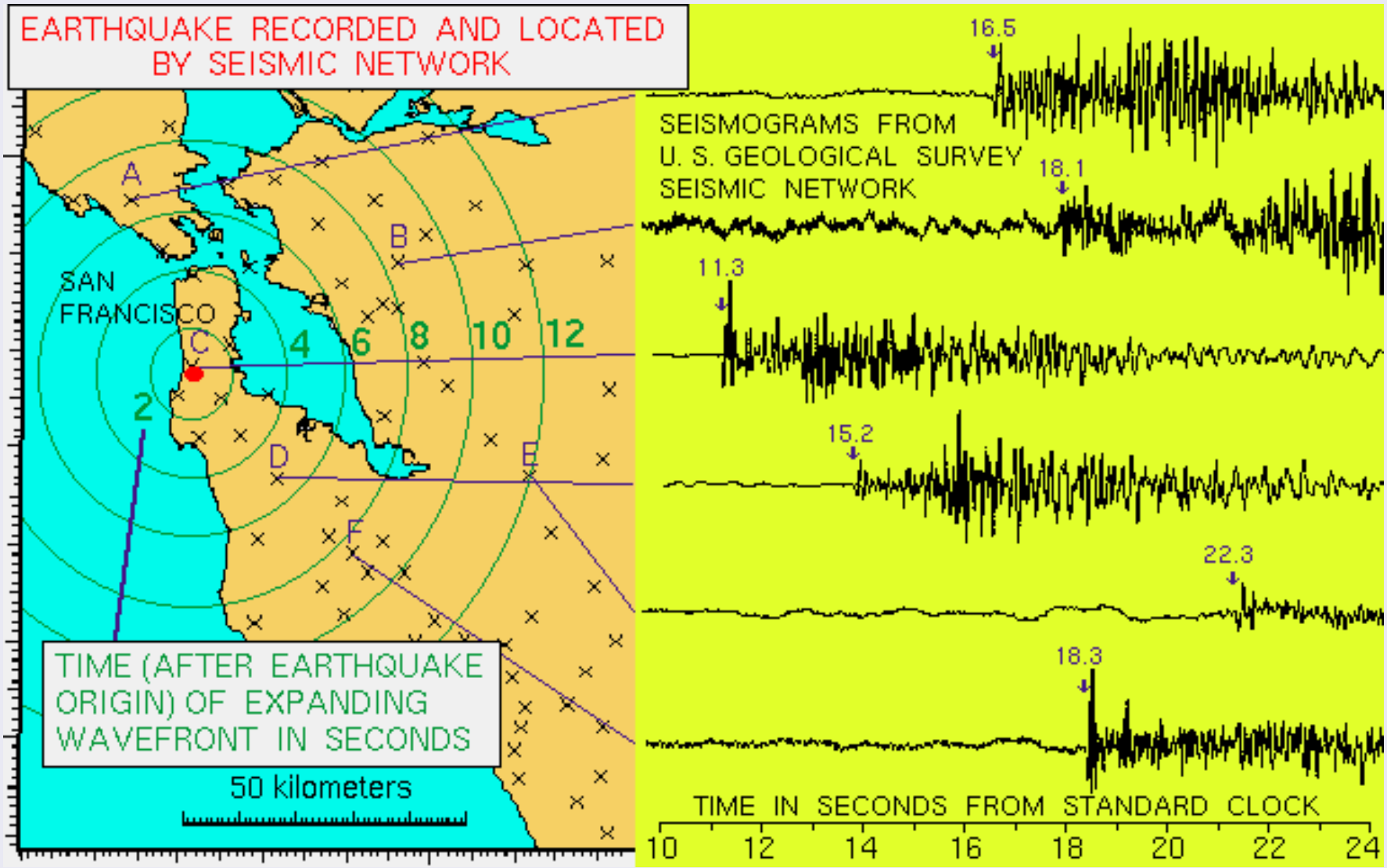
P Phase arrival detection: Detected by marking the time when the amplitude increases for the first time from zero, as the P Wave arrives first.

Secondary Wave (S Wave/Phase): This wave has an estimated velocity of 3.6 km/s and arrives later, as it is much slower than both Primary (P) Wave. It is a traverse wave, that is a wave that travels in a direction perpendicular to the direction of vibration, such wave consists of crest and trough, in terms of an amplitude.

S Phase arrival detection: It arrives later with a very large amplitude.

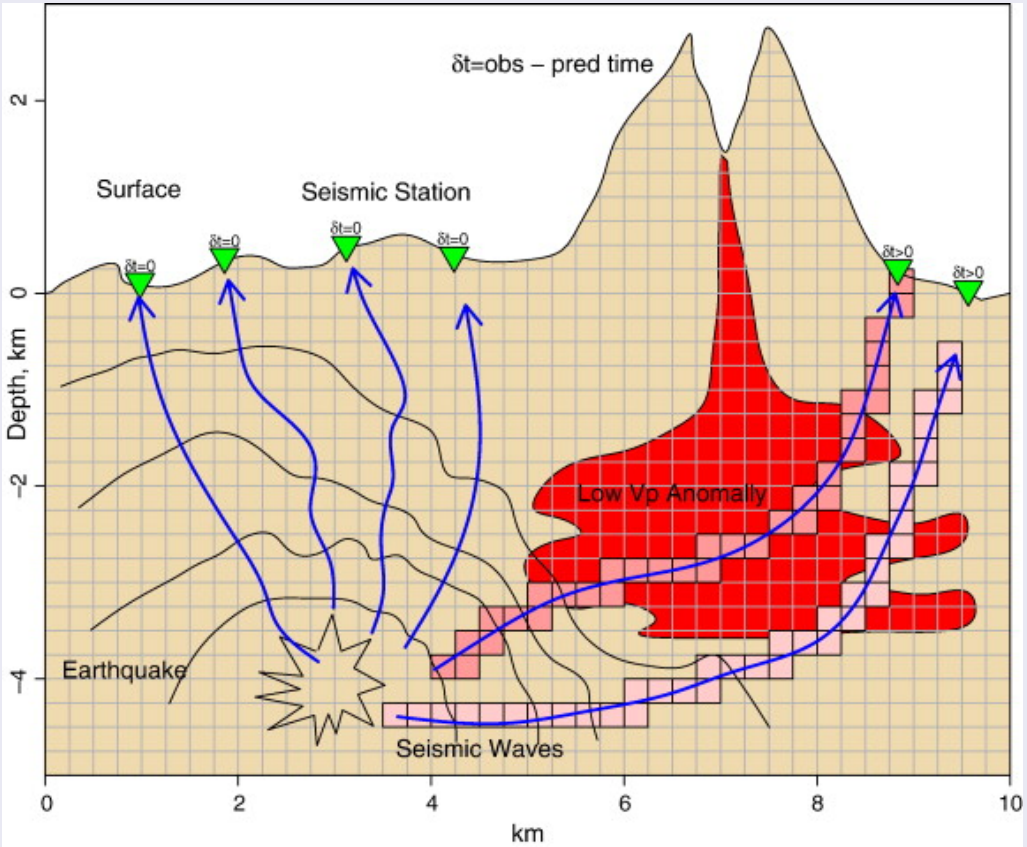
Example

P Phase Pickings of 6 seismographs (stations) from an Earthquake



Example

Cross Section of the 6 stations from an Earthquake



Each blue arrow: C_j is a ray path from the Hypocenter / Earthquake Focus (x_0, y_0, z_0) to the j^{th} Station (x_j, y_j, z_j).

Travel and Arrival Time Equations

Travel Time between the unknown Hypocenter and the j^{th} Station

Homogeneous 1-Layer Earth Crust (Velocity Model, V, is a constant):

$$T(x_j, y_j, z_j, x_0, y_0, z_0) = \frac{1}{V}\sqrt{(x_j - x_0)^2 + (y_j - y_0)^2 + (z_j - z_0)^2}$$

Heterogeneous Earth Crust ($V(x, y, z)$ is a 3D Velocity Model):

$$T(x_j, y_j, z_j, x_0, y_0, z_0) = \min \int_{C_j} \frac{ds}{V(x, y, z)}$$

where C_j is a ray path from the Hypocenter (x_0, y_0, z_0) to the j^{th} Station (x_j, y_j, z_j), and ds is the infinitesimal arc length. This is a line integral.

Calculated Arrival Time to predict the possible P Phase picking time

$$t_j^c = T(x_j, y_j, z_j, x_0, y_0, z_0) + t_0$$

Grid Search

Residual is defined as the difference between the observed (actual P Phase picking) and calculated arrival times: $r_j = t_j^o - t_j^c$

Types of sum of residuals

L2 Norm:

$$E_{L2} = \sum_{j=1}^{n} (r_i)^2 = \sum_{j=1}^{n} (t_j^o - t_j^c)^2$$

L2 Norm is suitable if we use derivative to minimise the sum of residuals.

L1 Norm:

$$E_{L1} = \sum_{j=1}^{n} |r_i| = \sum_{j=1}^{n} \left| t_j^o - t_j^c \right|$$

L1 Norm can deal with large outliers in data compared to L2 Norm. Moreover, the Grid Search does not need derivative, L1 Norm is preferred.

Grid Search

Root Mean Square (RMS)

$$RMS = \sqrt{\frac{E_{L2}}{n}}$$

This is just a square root of an average value of squared residuals, which will also be affected by outliers just like L2 Norm. L1 Norm is preferred.

Caution for using L2 Norm, L1 Norm or Root Mean Square (RMS)

RMS only gives an indication of the fit of the data, and a low RMS does not automatically mean an accurate hypocenter determination. Generally, the precision of the computational solution, which is based on various model assumptions, should not be mistaken as real accuracy of the location and origin time. Also, precision and accuracy do not imply each other.

Grid Search

Main Optimisation Problem (Algorithm) for the Grid Search

Minimise any type of sum of residuals (L2 Norm, L1 Norm or RMS).

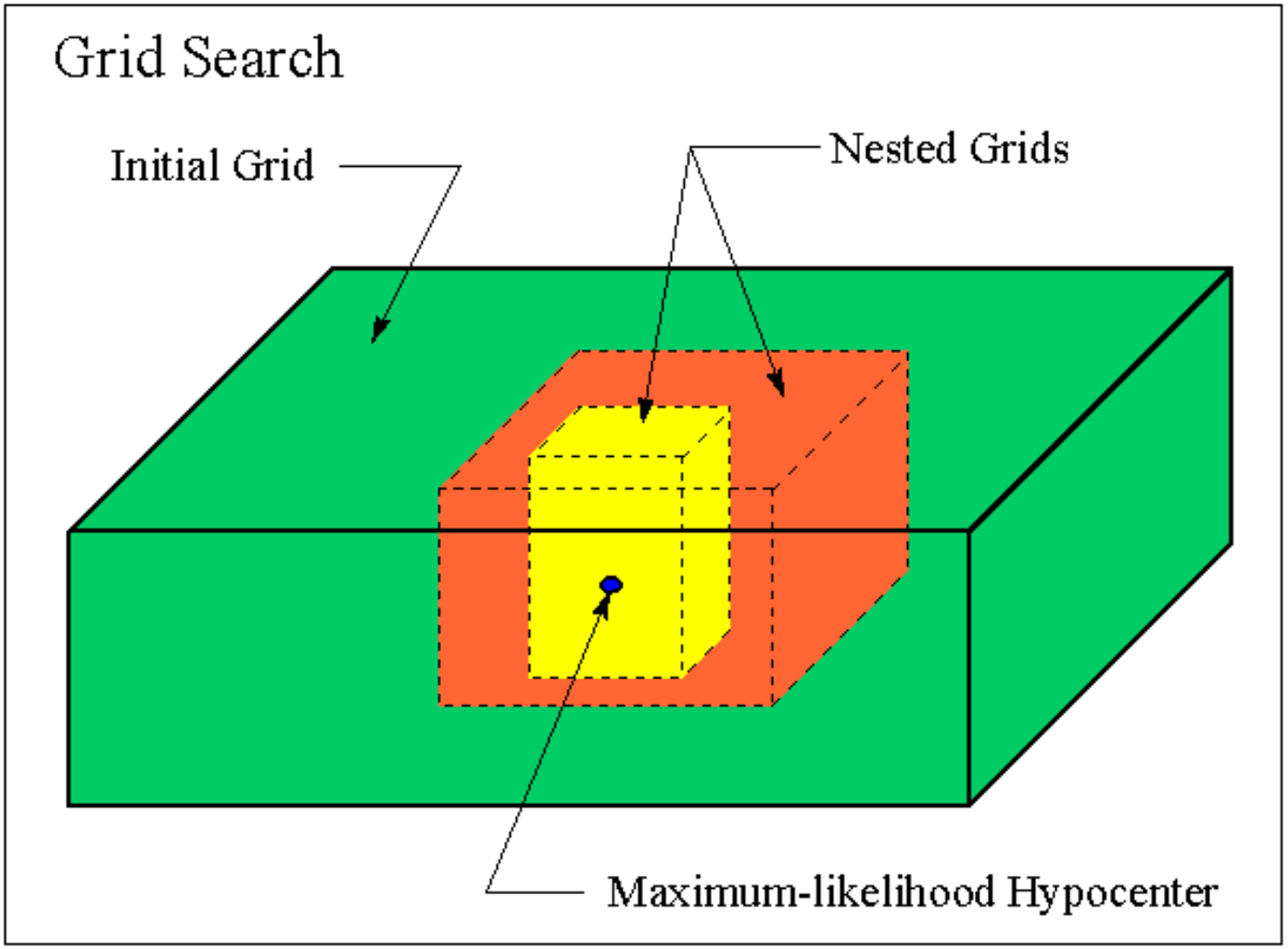
Subject to the known locations of stations, with each station (x_j, y_j, z_j) and multiple attempts to guess the appropriate value of the unknown Hypocenter (x_0, y_0, z_0) and Origin Time t_0.

The unknown Hypocenter (x_0, y_0, z_0) and Origin Time t_0 can be guessed by permutations of all possible values of (x_0, y_0, z_0, t_0), within a region, $[\min x_0, ..., \max x_0] \times [\min y_0, ..., \max y_0] \times [\min z_0, ..., \max z_0] \times [\min t_0, ..., \max t_0]$, with sampling intervals $(\Delta x_0, \Delta y_0, \Delta z_0, \Delta t_0)$, that is:

$$[\min x_0, (\min x_0) + \Delta x_0, ..., \max x_0] \times [\min y_0, (\min y_0) + \Delta y_0, ..., \max y_0] \times [\min z_0, (\min z_0) + \Delta z_0, ..., \max z_0] \times [\min t_0, (\min t_0) + \Delta t_0, ..., \max t_0]$$

Grid Search

Intuition, Visualisation and Illustration of Grid Search



Grid Search

Advantages of Grid Search

1. Good for complicated 3D Velocity Structure: Derivatives not required.

2. Systematic, deterministic coverage of search region.

3. Efficiently reads into memory 2D planes of 3D travel-time grid files.
3.1. Can be used for large number of observations and large 3D travel-time grids.

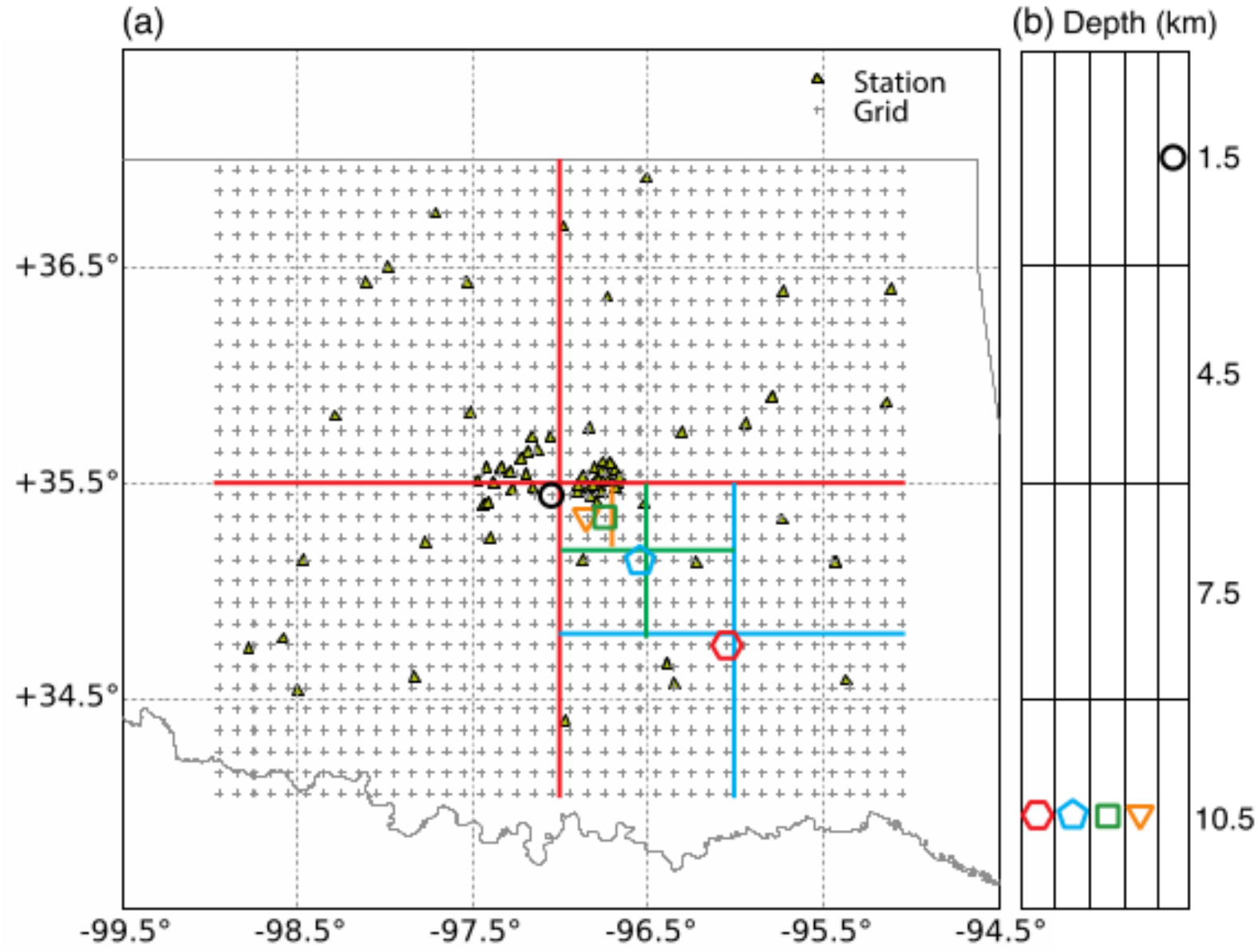
Disadvantages of Grid Search

1. Very time consuming (Computationally Expensive)

2. Final search grids may be too large (giving low resolution) or too small.

3. Requires careful selection of grid size and node spacing.

A Quick / Random Example of a Grid Search



Thank you very much for your time and attention!

References

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