Traveltime Adjoint Tomography

Tong Ping

Division of Mathematical Sciences, School of Physical and Mathematical Sciences
Asian School of the Environment, Nanyang Technological University, Singapore

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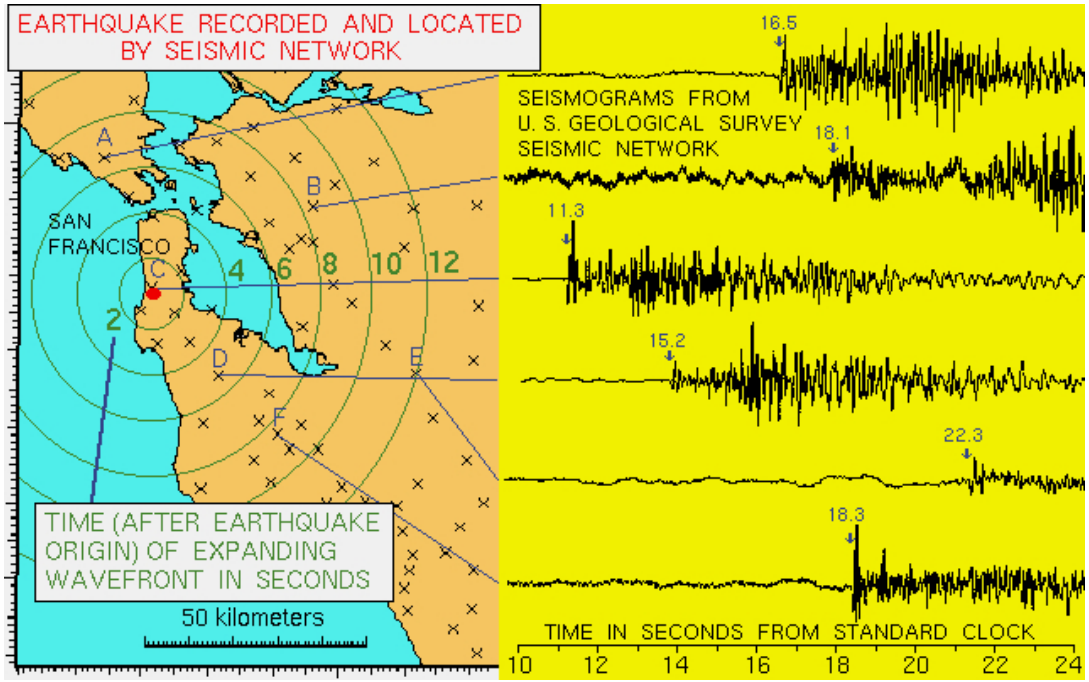
Outline

- Physical Models
- Traveltime tomography
- Mathematical Derivation of Traveltime adjoint tomography

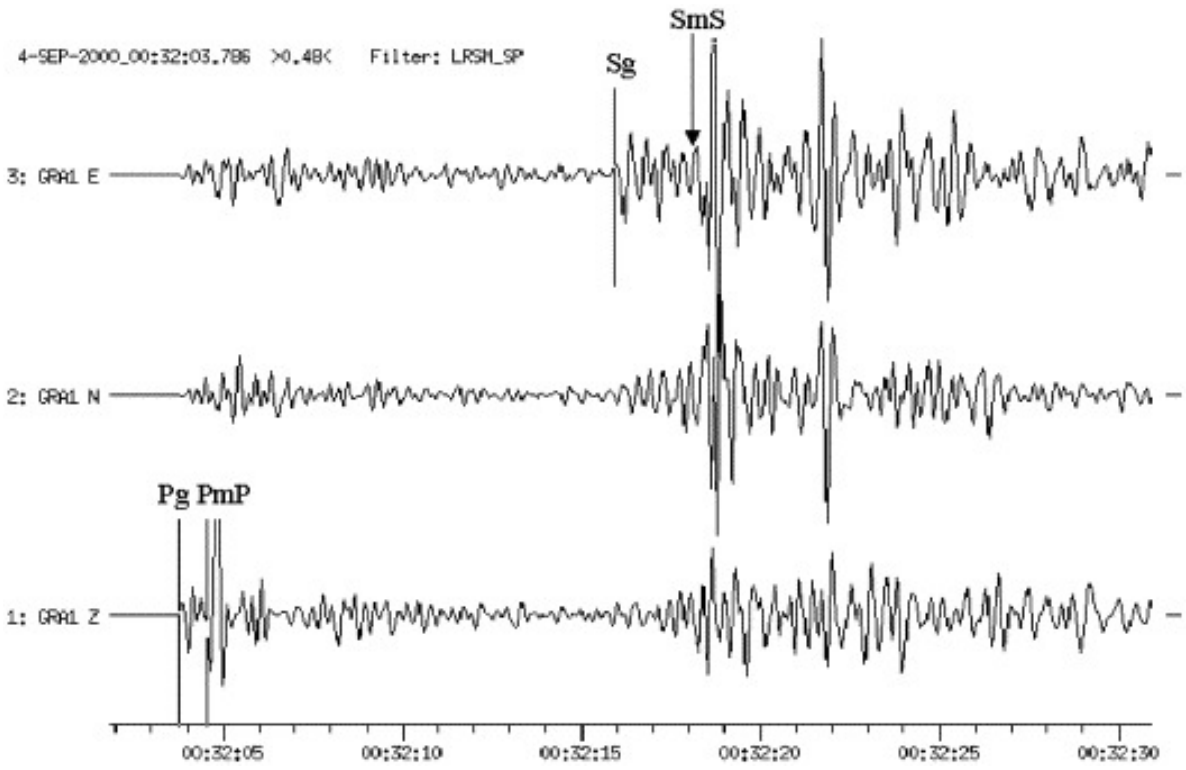
Data

- Arrival times: Ray-based inversions (Aki et al., JGR, 1976)
- Full/partial waveforms: Wave-equation based inversions (Tromp et al., GJI, 2005; Tong et al., Solid Earth, 2014a,b)



Data

Reflected phases in data



Physical Models for Traveltime

Traveltime is the most robust information that can be extracted from seismic records.

- Integral Form

$$t = \int_L \frac{1}{v(\mathbf{x})} dl = \int_L s(\mathbf{x}) dl$$

- Wave Equation

$$\partial_t^2 u(t, \mathbf{x}) - c^2(\mathbf{x}) \Delta u(t, \mathbf{x}) = f(t - \tau) \delta(\mathbf{x} - \mathbf{x_s}), \quad \mathbf{x} \in \Omega.$$

- Differentiation Form

$$\nabla T(\mathbf{x}) \cdot \nabla T(\mathbf{x}) = \frac{1}{v^2(\mathbf{x})} = s^2(\mathbf{x})$$

Traveltime tomography

$$\Delta t = T_{\text{obs}} - T_{\text{cal}} = \int_{\Omega} K(\mathbf{x}) \left[v_{\text{true}}(\mathbf{x}) - v(\mathbf{x}) \right] d\mathbf{x}$$

- Integral Form

$$K(\mathbf{x}) = -\frac{1}{c^2(\mathbf{x})} \delta(\mathbf{x} - \mathbf{x}_l)$$

- Wave Equation

$$K(\mathbf{x}) = \int_0^T \left[2v^2(\mathbf{x}) \nabla q(\mathbf{x}, t) \cdot \nabla u(\mathbf{x}, t) \right] dt$$

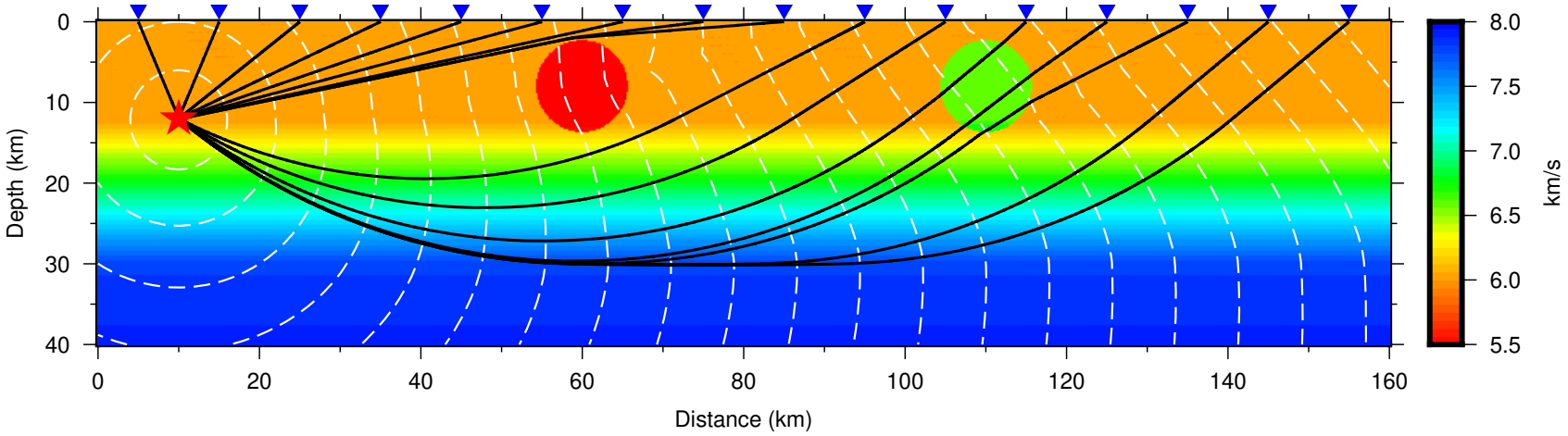
- Differentiation Form

$$K(\mathbf{x}) = \frac{P(\mathbf{x})}{v(\mathbf{x})}.$$

Ray-based traveltime tomography

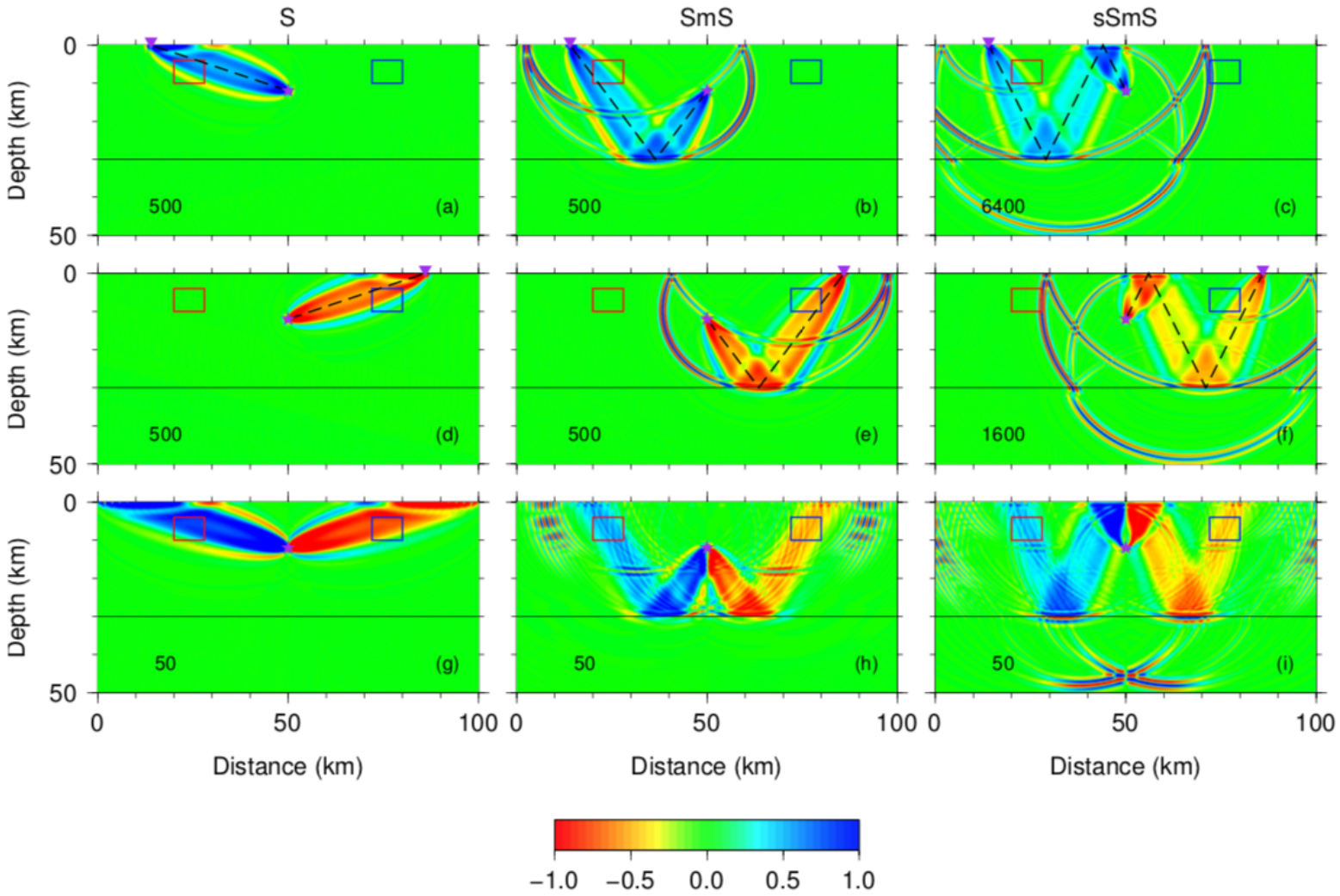
- The ray paths can be efficiently and accurately determined by solving the following eikonal equation with the advanced multistencil fast marching method (Rawlinson et al., 2004) or the fast sweeping method (Zhao 2004).

$$c(\mathbf{x})|\nabla T(\mathbf{x})| = 1.$$



Wave equation-based traveltime tomography

- Numerical methods are required to solve both the forward and adjoint wave equations (Tromp et al. 2005; Tong et al., 2014).



Traveltime adjoint tomography

- Numerical methods are required to solve both the forward and adjoint eikonal equations (Sei and Symes 1994, Leung and Qian 2006).