Joint Epicentre Determination

THIS communication describes a method of determining station travel time corrections and the positions and origin times of more than one earthquake simultaneously. Application of the method to earthquakes from the same region should reveal any regional bias in travel times. Some preliminary results are presented.

Suppose the rough epicentre, depth and origin time of a seismic event are known, the equation of condition for calculating the corrections to these approximate values is[1]

$$\delta H + \delta h \frac{\partial T}{\partial h} + x \cos \alpha_j \frac{\partial T}{\partial \Delta_j} - y \sin \alpha_j \frac{\partial T}{\partial \Delta_j} = \delta T_j \qquad (1)$$

where $\delta T_j = A_j - H - T_j$; H is the approximate origin time of the event; h is the approximate depth of the event; Δ_j is the distance from the approximate epicentre to station j; α_j is the azimuth from the approximate epicentre to station j; A_j is the time of arrival (of the P waves) at station j; T_j is the travel time (of the P waves) from the approximate epicentre to station j; $\partial T/\partial \Delta_j$ is the partial derivative of the travel time T $(= f(\Delta_j, h))$ with respect to distance at the point Δ_j, h; $\partial T/\partial h$ is the partial derivative of the travel time T with respect to depth at Δ_j, h; T, $\partial T/\partial \Delta_j$, and $\partial T/\partial h$ are obtained from travel time tables.

The unknowns x, y, δh and δH, the corrections to latitude, longitude, depth and origin time respectively, can be estimated by the method of least squares from equation (1) provided $j > 4$. A more accurate estimate of epicentre, depth and origin time can thus be obtained. Further corrections to this new epicentre can be calculated in the same way and the process repeated calculating successively better approximations to the (least squares) estimates of the epicentre parameters until the corrections become small enough to neglect. Convergence towards the best estimates is usually rapid.

Recent work (for example, ref. 2) has shown that equation 1 should include a term for the station correction S_j, where S_j is the difference between the observed travel time and the travel time obtained from the travel time tables. The results of several special studies have been published giving estimates of station corrections.

When these station corrections are available they can be applied to the arrival times A_j and these corrected times substituted in equation 1. The epicentre calculation can then proceed as before.

Cleary and Hales[3] have also shown that some station corrections are dependent on azimuth, and Herrin and Taggart[4] and Helterbran and Jordan[5] report that, at least from the LONGSHOT area, travel times vary azimuthally. Corrections for the azimuthal variations should therefore also be applied if accurate epicentres are to be estimated. But these corrections, particularly the source bias, are difficult to detect, because the usual method of estimating epicentres, by definition, shifts the position of the event until the residuals show no bias. Some of the bias can, however, be detected by estimating simultaneously the station corrections and epicentres of several events from the same region.

Equation 1 can be rewritten to include the station corrections S_j; S_j contains the travel time bias as well as the station effect. Thus if δS_j is the correction to the approximate value of S_j, equation 1 becomes for the station j recording the ith event

$$\delta S_j + \delta H_i + \delta h_i \frac{\partial T}{\partial h_i} + x_i \cos \alpha_{ij} \frac{\partial T}{\partial \Delta_{ij}} - y_i \sin \alpha_{ij} \frac{\partial T}{\partial \Delta_{ij}} = \delta T_{ij} \qquad (2)$$

where $\delta T_{ij} = A_{ij} - H_i - T_{ij} - S_j$.

As equation 2 stands, δS_j and δH_i are linearly dependent and equation 2 cannot be used to estimate the unknowns. This difficulty can be overcome by assigning a value to one S_j or, what is probably better, by assuming that $\Sigma_j S_j = 0$, that is, it is assumed that the mean station correction is zero. With this assumption the epicentre, depth, origin time and station corrections can be estimated—at least in theory.

A computer programme has been written for joint epicentre and station correction estimation, based on equation 2. Preliminary tests using seven events from the Aleutian Islands recorded at some or all of thirty stations (each station recorded at least two events) show rapid convergence (computed corrections are usually < 0.01 after three or four iterations) if the depth of at least one event is held fixed (restrained). With no depth restraints convergence is slow and the estimates tend to oscillate.

The events used include the LONGSHOT explosion: the normal method of epicentre determination gives estimates 25–30 km north of the true epicentre. The method of joint epicentre determination should give a more accurate estimate; LONGSHOT is therefore a valuable datum against which to test the joint epicentre method.

To obtain the results shown in Tables 1 and 2 all events have had their depth restrained to the depth estimated by the US Coast and Geodetic Survey; the event of October 29, 1965, LONGSHOT, has been restrained to 0 km. The arrival times used for these calculations are taken from the *Earthquake Data Reports* published by the US Coast and Geodetic Survey. The stations used have been chosen to cover as many azimuths as possible: all stations closer than 15° to any of the epicentres have been excluded from the analysis as have all stations for which the residuals after convergence were greater than 3 sec (the epicentres were recalculated with the stations with residuals greater than 3 sec removed).

Table 1 shows for each of the seven events: (1) the US Coast and Geodetic Survey epicentres (the LONGSHOT epicentre is the true epicentre), (2) the epicentre determined by the standard method using only those stations used in the joint determinations and (3) the results of the joint epicentre determination; Table 2 shows the station corrections. It is clear from Table 1 that the joint epicentre method gives a much closer estimate (≈ 1 km) to the true LONGSHOT epicentre than the normal method (≈ 23 km).

Table 1

(1) Komandorsky Isles: July 19, 1966			
USCGS epicentre	56·20 N. 164·90 E.	$h = 18$ km	$H = 1:40:53·9$
Epicentre using thirty stations	56·43 N. 164·46 E.	$h = 18$ km*	$H = 1:40:54·9$
Epicentre using joint method	56·28 N. 164·62 E.	$h = 18$ km*	$H = 1:40:54·8$
(2) Rat Islands, Aleutian Islands: June 2, 1966			
USCGS epicentre	51·08 N. 175·97 E.	$h = 41$ km*	$H = 3:27:53·3$
Epicentre using thirty stations	51·04 N. 175·89 E.	$h = 41$ km*	$H = 3:27:52·9$
Epicentre using joint method	50·92 N. 175·98 E.	$h = 41$ km*	$H = 3:27:53·2$
(3) Rat Islands, Aleutian Islands (LONGSHOT): October 29, 1965			
True epicentre	51·44 N. 179·18 E.	$h = 0$ km*	$H = 21:0:0·1$
Epicentre using thirty stations	51·65 N. 179·13 E.	$h = 0$ km*	$H = 20:59:56·9$
Epicentre using joint method	51·45 N. 179·18 E.	$h = 0$ km*	$H = 20:59:56·8$
(4) Andreanof Islands, Aleutian Islands: July 19, 1966			
USCGS epicentre	51·73 N. 173·30 W.	$h = 47$ km*	$H = 19:20:33·4$
Epicentre using thirty stations	51·81 N. 173·35 W.	$h = 47$ km*	$H = 19:20:33·5$
Epicentre using joint method	51·75 N. 173·40 W.	$h = 47$ km*	$H = 19:20:33·9$
(5) Fox Islands, Aleutian Islands: August, 11, 1966			
USCGS epicentre	52·76 N. 169·74 W.	$h = 61$ km	$H = 10:45:59·6$
Epicentre using thirty stations	52·76 N. 169·77 W.	$h = 61$ km*	$H = 10:45:59·1$
Epicentre using joint method	52·71 N. 169·79 W.	$h = 61$ km*	$H = 10:45:59·6$
(6) South of Alaska: February 6, 1965			
USCGS epicentre	53·29 N. 161·81 W.	$h = 33$ km*	$H = 16:50:28·6$
Epicentre using thirty stations	53·24 N. 161·88 W.	$h = 33$ km*	$H = 16:50:28·2$
Epicentre using joint method	53·06 N. 161·92 W.	$h = 33$ km*	$H = 16:50:28·2$
(7) South of Alaska: January 22, 1966			
USCGS epicentre	55·97 N. 153·69 W.	$h = 33·0$ km*	$H = 14:27:7·9$
Epicentre using thirty stations	55·96 N. 153·89 W.	$h = 33·0$ km*	$H = 14:27:7·8$
Epicentre using joint method	55·83 N. 153·93 W.	$h = 33·0$ km*	$H = 14:27:7·7$

* Restrained parameters.

Table 2

Station	Azimuth	Station correction	LONGSHOT residual Observed $-J$–B time
MBC	23·0	−0·7	Not available
COL	41·0	−1·5	−3·6
WES	51·8	−0·9	−5·1
OTT	52·3	−1·8	−5·9
SJG	62·8	−1·0	−4·2
CPO	66·0	−1·9	−5·0
BMO	77·9	+0·0	−2·7
UBO	78·1	+0·5	−1·7
PAS	90·9	−0·3	−2·4
KIP	148·1	+2·9	−0·6
PPT	152·8	+1·9	−1·9
VUN	184·5	+2·4	−2·0
KOU	198·0	+0·8	−1·8
HNR	205·0	+1·2	−2·5
RIV	206·0	+2·2	−1·9
BRS	206·6	+0·0	−2·5
TOO	209·1	+2·0	−2·6
CTA	214·8	−1·1	−3·5
PMG	217·9	−0·4	−2·9
DAR	231·7	−0·1	−2·9
MAT	259·4	−0·0	−2·5
SHL	286·6	−1·4	−3·6
QUE	307·8	−0·1	−2·5
TEH	322·0	+0·4	−3·1
IST	340·4	−0·5	−5·4
UPP	352·0	−1·1	−5·1
TRO	353·4	−0·1	−4·0
CLL	353·5	−0·6	Not available
STU	355·7	−0·4	Not available
KON	356·3	−0·7	−4·2

The error in the LONGSHOT epicentre as determined by the standard method is caused by a regional bias in travel time in the Aleutian region. This bias can be seen in the LONGSHOT residuals—the difference between the observed travel times and J–B travel times from the true LONGSHOT epicentre (Table 2)—superimposed on a base-line shift of about −3 sec. The station corrections (Table 2) computed by considering seven Aleutian events together reveal a similar bias (but not of course the base-line shift which is taken up in adjustments to the origin time): stations to the south of the Aleutian Islands show generally positive station corrections; those to the north show generally negative corrections. The joint epicentre method takes account of the bias.

Thus in a region of known bias the method of joint epicentre determination has detected the bias and produced a more accurate estimate for a (known) explosion epicentre than can be obtained by the normal method. To do this the depths of all the events were assumed to be known. Errors in these assumed depths will tend to bias the epicentral estimates but this bias is probably small. The accuracy of the estimates of the LONGSHOT epicentre suggests either that the estimates of depth are accurate or, more probably, that only gross errors in depth have any appreciable effect on the epicentral estimates.

The joint epicentre method as outlined here breaks down if all the events used are close together—that is, if they are spread over only a few degrees of arc—because the station corrections and travel times are virtually linearly dependent. The method can, however, be adapted to obtain the relative positions of a group of closely spaced events by restraining the epicentre and time of origin of one of the events; the station corrections are then the only unknowns in the equations of condition of the restrained event. If the absolute position of the reference event is known or can be estimated by considering the reference event as one of a group of more widely spaced events—as illustrated here for the seven Aleutian Island events—epicentres determined relative to the reference event will be estimates of the absolute epicentres. A detailed picture of events on both the local and regional scale can therefore be built up. (In theory all events from a given area could be considered at once; in practice the number of events that can be considered at any one time is limited by the storage capacity of the computer available.)

More tests are required to prove the method of joint epicentre determination and work is continuing, particularly to study the value of the method for focal depth estimation and to calculate the confidence limits on the estimates. This communication simply draws attention to the method.

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Pulsating Radio Auroral Echoes

PULSATING auroral echoes have been described[1] which accompany sudden commencements, and attention has been directed to the fact that the periods of repetition of these echoes are close to those predicted for the bounce periods of hydromagnetic waves along the geomagnetic line of force terminating in the echo region. More exactly, the bounce periods considered are those for waves having angular frequencies ω which are considerably less than Ω_0 the minimum (equatorial plane) particle cyclotron frequency along the field line. Thus there can be an apparent agreement between echo period and hydromagnetic bounce period for $\omega \simeq 0$ up to $\omega \simeq 0{\cdot}1\ \Omega_0$, beyond which the travel time increases as $\omega \rightarrow \Omega_0$.

Other workers[2-4] have tried to relate periodicities in absorption and X-ray measurements to low frequency standing-wave oscillations of the magnetic field lines and it has been suggested that the particle beam giving rise to these effects is modulated by large-scale variations in the value of the magnetospheric magnetic field. If the interpretations in ref. 1 are correct, particularly where the special event of July 15, 1959, is concerned, then the mechanism just described would be of too large a scale perpendicular to the field lines to allow the resolution in range which is obtained.

Another possibility considered here is that the pulsations arise from a resonant interaction of the type discussed by Wentzel[5], and, because we require periodicities related to hydromagnetic bounce periods, we are restricted to particle beams capable of interacting with left-hand polarized waves for which $\omega \not> \Omega_{0i}$, where Ω_{0i} is the minimum proton gyrofrequency on the path. The conditions for resonance are[6]

$$\left|\frac{\Omega}{\omega}\right| = \left|\frac{V_z}{U}\right| + 1 \qquad (1)$$

for particles and waves of similar polarization but opposite directions along the field;

$$\left|\frac{\Omega}{\omega}\right| = \left|\frac{V_z}{U}\right| - 1 \qquad (2)$$

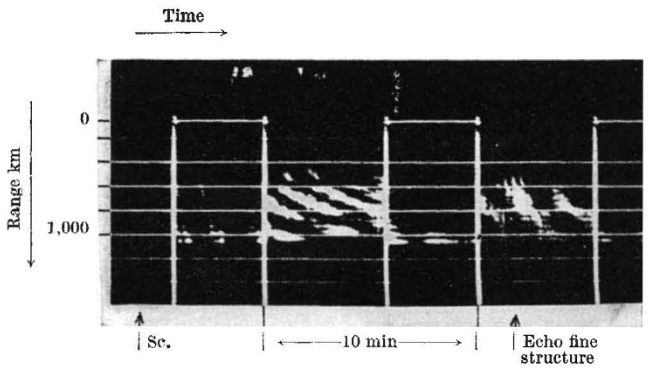

Fig. 1. Event on 8-5-60, 0421 U.T., showing fine structure in an echo pulse. (Radar operating on two aerials.)