Adjoint state-based earthquake location

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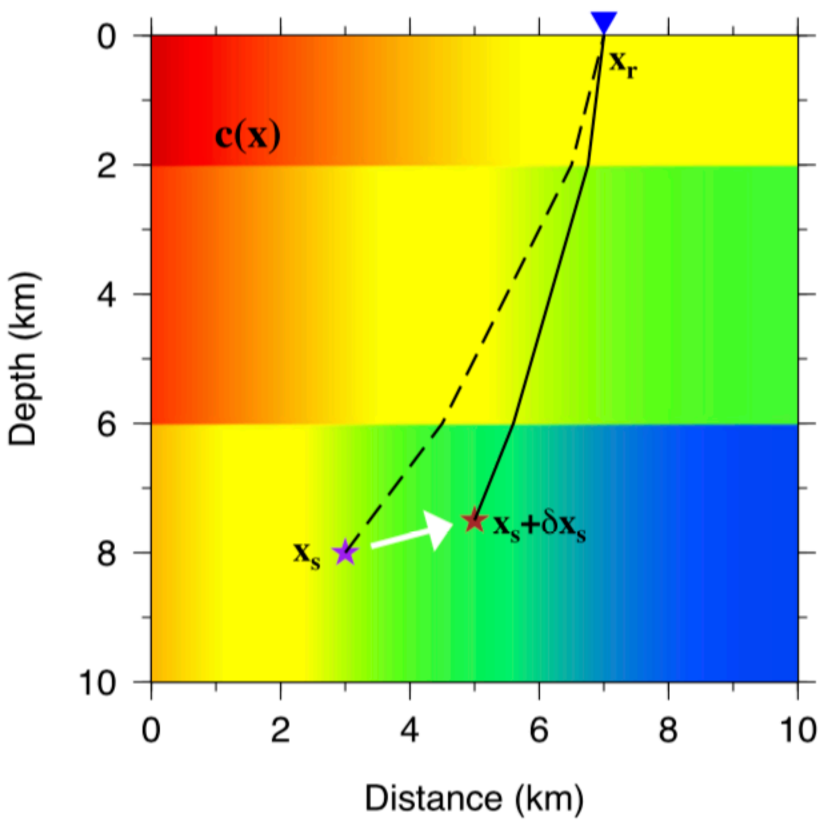
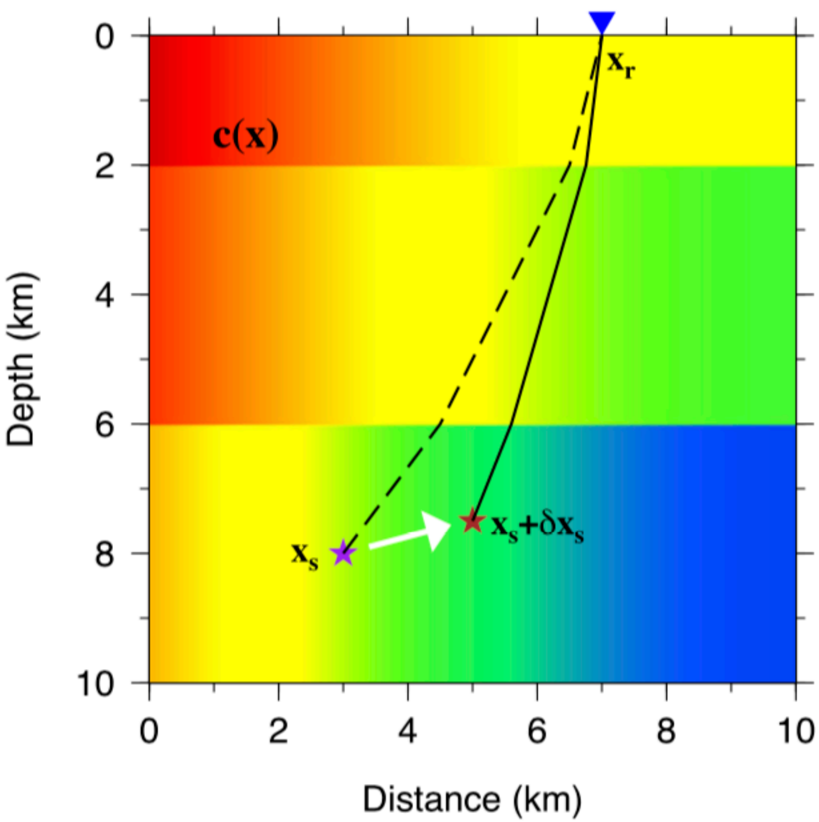
Ray-based earthquake location



δx_s ----- > δt fixed c(x)

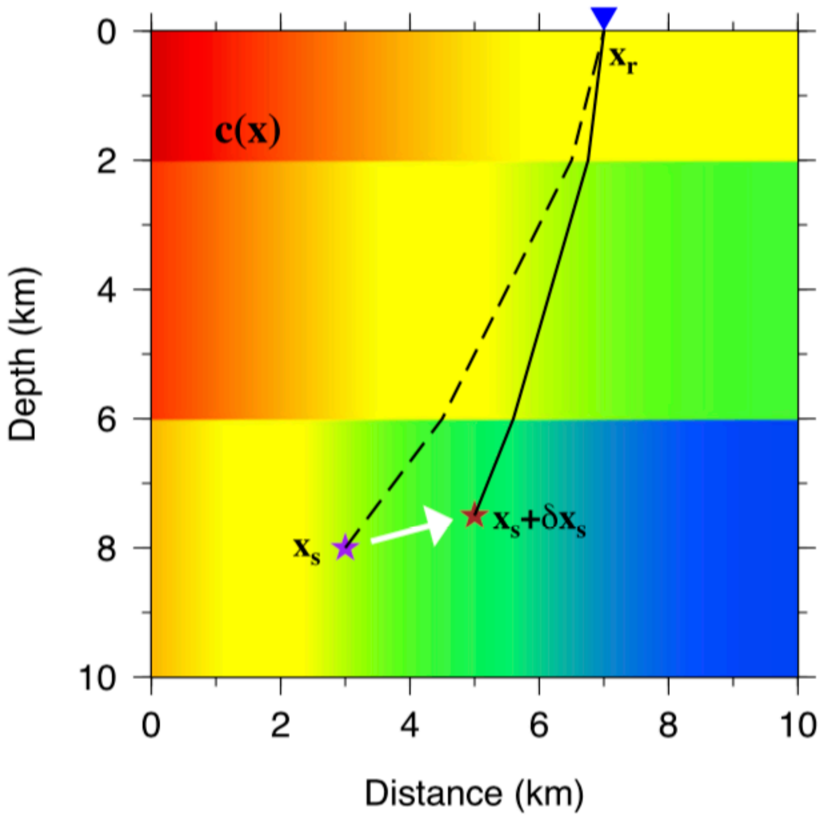
Ray-based earthquake location



$$\delta t = \int K_m(\boldsymbol{x})\, \delta \ln m(\boldsymbol{x}) \delta(\boldsymbol{x} - \boldsymbol{x}_s)\, dl$$

$$m(x) = (t, x_s)$$

Ray-based earthquake location



$$K_{t_0}(x) = -1$$

$$K_{\theta_s}(x) = \frac{(R-h)\, sin\, \mathrm{i}\, cos\, \alpha\, \theta_s}{V_{x_s}}$$

$$K_{\varphi_s}(x) = -\frac{(R-h)\, sin\, \mathrm{i}\, sin\, \theta\, sin\, \alpha\, \varphi_s}{V_{x_s}}$$

$$K_{z_s}(x) = -\frac{cos\, i\, z_s}{V_{x_s}}$$

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Adjoint state-based earthquake location

1）*The relationship between δt and δu*

The cross-correlation between synthetic data and observed data is:

$$C(\tau) = \int_{t_1}^{t_2} u(t-\tau)u_{obs}(t)dt = \int_{t_1}^{t_2} u(t-\tau)[u(t)+\delta u(t)]dt$$

Do Taylor expansion for $u(t-\tau) = u(t) - \tau\frac{\partial u(t)}{\partial t} + \frac{1}{2}\tau^2\frac{\partial^2 u(t)}{\partial t^2} + \cancel{o(\tau^2)}$

We have

$$C(\tau) = \int_{t_1}^{t_2} [u(t) - \tau\frac{\partial u(t)}{\partial t} + \frac{1}{2}\tau^2\frac{\partial^2 u(t)}{\partial t^2}]u(t)dt + \int_{t_1}^{t_2} [u(t) - \tau\frac{\partial u(t)}{\partial t} + \cancel{\frac{1}{2}\tau^2\frac{\partial^2 u(t)}{\partial t^2}}]\delta u(t)dt$$

Adjoint state-based earthquake location

1）*The relationship between* δt *and* δu

$$\partial_\tau C(\tau) = \int_{t_1}^{t_2} [-\frac{\partial u(t)}{\partial t} + \tau \frac{\partial^2 u(t)}{\partial t^2}] u(t) dt + \int_{t_1}^{t_2} -\frac{\partial u(t)}{\partial t} \delta u(t) dt = 0$$

$$\int_{t_1}^{t_2} -\frac{\partial u(t)}{\partial t} u(t) dt = u^2(t_2) - u^2(t_1) = 0 = u(t_1) = u(t_2)$$

So, we get the the time shift due to the displacement perturbation:

$$\delta \mathrm{t} = \tau = \frac{\int_{t_1}^{t_2} \dot{u}(t) \delta u(t) dt}{\int_{t_1}^{t_2} \ddot{u}(t) u(t) dt} = \frac{<\dot{u}(t),\ \delta u(t)>_t}{<\ddot{u}(t),\ u(t)>_t}$$

The above mathematical derivation does not involve in ***wave equation theory***

Adjoint state-based earthquake location

2）*The relationship between* δu *and* δf

$$\rho \frac{\partial^2 u}{\partial t^2} - \nabla \cdot [T + \delta T] = f$$

The solution:

$$u = G \otimes f$$

$$u + \delta u = (G + \delta G) \otimes (f + \delta f)$$

We get the difference:

$$\delta u = G \otimes \delta f + \delta G \otimes f \approx G \otimes \delta f$$

Adjoint state-based earthquake location

3）*Calculate the corresponding kernel of source*

$$\delta\chi = \sum_{r=1}^{N_r} [t_{sr} - t_{sr}^{obs}] \delta t_{sr}$$

Using the following relationship

$$\delta t = \frac{< \dot{u}(t), \quad \delta u(t) >_t}{< \ddot{u}(t), \quad u(t) >_t}$$

$$\delta u = G \otimes \delta f$$

So,

$$\begin{aligned} \delta\chi &= \sum_{r=1}^{N_r} [t_{sr} - t_{sr}^{obs}] \frac{<\dot{u}(t),\ \delta u(t)>_t}{<\ddot{u}(t),\ u(t)>_t} \\ &= \sum_{r=1}^{N_r} [t_{sr} - t_{sr}^{obs}] \frac{<\dot{u}(t),\ G \otimes \delta f>_t}{<\ddot{u}(t),\ u(t)>_t} \end{aligned}$$

Adjoint state-based earthquake location

3）*Calculate the corresponding kernel of source*

$$\delta\chi = \sum_{r=1}^{N_r}[t_{sr} - t_{sr}^{obs}]\frac{<\dot{u}(t),\ G\otimes\delta f>_t}{<\ddot{u}(t),\ u(t)>_t}$$

Since we have:

$$< h,\ G \otimes f >_t = < G^* \otimes h,\ f >_t$$

$$G^*(t) = G(-t)$$

we further get:

$$\delta\chi = \sum_{r=1}^{N_r}[t_{sr} - t_{sr}^{obs}]\frac{<G^*\dot{\otimes}u(t),\ \delta f>_t}{<\ddot{u}(t),\ u(t)>_t}$$

Adjoint state-based earthquake location

3）*Calculate the corresponding kernel of source*

$$\delta\chi = \sum_{r=1}^{N_r}[t_{sr} - t_{sr}^{obs}]\frac{<G^{*}\otimes\dot{u}(t),\ \delta f>_t}{<\ddot{u}(t),\ u(t)>_t}$$

The adjoint source:

$$f^{+} = \sum_{r=1}^{N_r}[t_{sr} - t_{sr}^{obs}]\frac{\dot{u}(t)}{<\ddot{u}(t),\ u(t)>_t}$$

The adjoint wavefield:

$$s^{+} = G(-t)\otimes f^{+}$$

So the variation of cost function:

$$\delta\chi = < s^{+},\ \delta f >_{t}$$

Adjoint state-based earthquake location

3) *Calculate the corresponding kernel of source*

$$f\,(x,y,t) = h(t) * \delta(x - x_s) * \delta(y - y_s)$$

$$h(t) = (-2\alpha^3/\sqrt{\pi}(t - t_s)\exp(-\alpha^2(t - t_s)^2))$$

Taking the variation of $f(x,y,t)$, *we get*

$$\delta f\,(x,y,t) = -\dot{h}(t) * \delta(x - x_s) * \delta(y - y_s) * \delta t_s +$$

$$h(t) * \partial_{x_s}[\delta(x - x_s) * \delta(y - y_s)] * \delta x_s +$$

$$h(t) * \partial_{y_s}[\delta(y - y_s) * \delta(y - y_s)] * \delta y_s$$

$$\delta\chi = < s^+,\ \delta f >_t = \int_0^T \int_\Omega \delta f(x,y,t) s^+(x,y,T-t)dxdydt$$

Adjoint state-based earthquake location

3）*Calculate the corresponding kernel of source*

$$\delta f\ (x, y, t) = -\dot{h}(t) * \delta(x - x_s) * \delta(y - y_s) * \delta t_s +$$

$$h(t) * \partial_{x_s}[\delta(x - x_s) * \delta(y - y_s)] * \delta x_s +$$

$$h(t) * \partial_{y_s}[\delta(y - y_s) * \delta(y - y_s)] * \delta y_s$$

Then, recalling the cost function

$$\delta\chi = < s^+,\ \delta f >_t$$

$$= \int_0^T \int_\Omega \delta f(x, y, t) s^+(x, y, T - t) dxdydt$$

$$= - \int_0^T \dot{h}(t) * s^+(x_s, y_s, T - t)\, dt * \delta t_s + \int_0^T h(t) * \partial_{x_s} s^+(x_s, y_s, T - t) dt * \delta x_s +$$

$$\int_0^T h\ (t) * \partial_{y_s} s^+(x_s, y_s, T - t)\, dt * \delta y_s$$

Adjoint state-based earthquake location

3）*Calculate the corresponding kernel of source*

$$\delta\chi = \langle s^+,\ \delta f \rangle_t$$

$$= -\int_0^T \dot{h}(t) * s^+(x_s, y_s, T-t)dt * \delta t_s + + \int_0^T h(t) * \partial_{x_s} s^+(x_s, y_s, T-t)dt\ \delta x_s +$$

$$\int_0^T h\,(t) * \partial_{y_s} s^+(x_s, y_s, T-t)dt\ \delta y_s$$

$$k_{t_s} = -\int_0^T \dot{h}(t) * s^+(x_s, y_s, T-t)\,dt$$

$$k_{x_s} = \int_0^T h\,(t) * \partial_{x_s} s^+(x_s, y_s, T-t)dt$$

$$k_{y_s} = \int_0^T h\,(t) * \partial_{y_s} s^+(x_s, y_s, T-t)dt$$

Thanks