Topic 10:
Price Discrimination

EC 3322

Semester I – 2008/2009

Introduction

- **Price discrimination** → the use of **<u>non-uniform pricing</u>** to **max. profit**:
  - Charging consumers different prices for the same product.
  - Charging a consumer a price which varies with the quantity bought.
- **Not all price differences reflect price discrimination** → cost differential of supplying the products to different group of consumers.
- <u>Examples</u>:
  - Magazines, movie or museums tickets sold at **normal price** and **concession price** for students and senior citizens.
  - The use of **rebate coupons**.
  - Prescription drugs, music CDs, or movie DVDs are **cheaper in some countries**. Gasoline price in Singapore and in Johor.
  - **Business** and **first class** travel vs. **economy class**.

Introduction …

- Why price discrimination is profitable? → because **consumers have different valuations** (willingness-to-pay/ WTP) → those with higher valuations pay more → in contrast to the uniform pricing.

- Conditions for **successful price discrimination** policy:

  - The firm adopting the policy must have some **market power** (able to set p>MC).

  - The firm must be able to **distinguish consumers** on the basis of their WTP → this WTP must vary across consumers and (or) units purchased.

  - The firm must be able to **prevent arbitrage or resale** from consumers who pay at a lower price to consumers who are willing to pay a higher price.

Price Discrimination

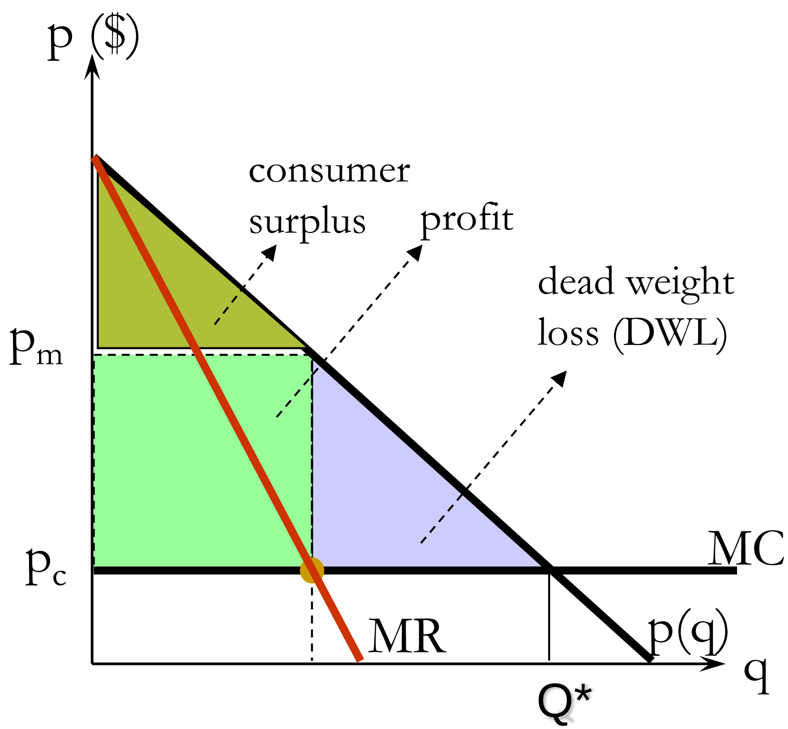
- There are three different types of price discrimination:
  - **First-degree** price discrimination (or personalized pricing) → Each consumer pays a different price depending on the WTP → consumers are left with no consumer surplus.
  - **Second-degree** price discrimination (or menu pricing) → the price per unit depends on the number of units purchased → the firm is not able to capture all consumer surplus.
  - **Third-degree** price discrimination (group pricing) → each group of consumers faces its own price per unit → the firm is not able to capture all consumer surplus.

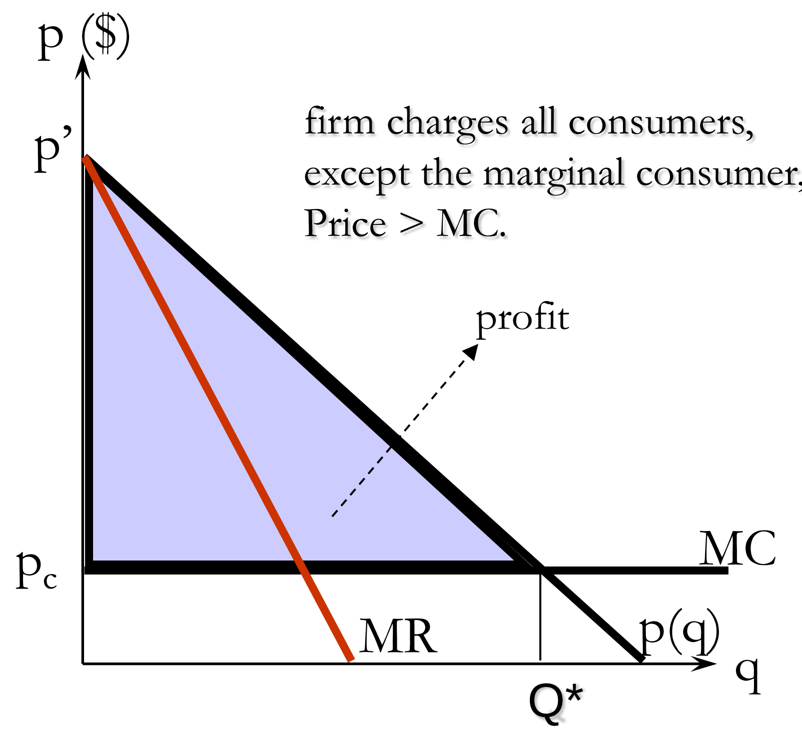
Price Discrimination …

- Specific examples of price discrimination strategies:
  - **Two-Part Tariff →**: The firm charges a **lump-sum fee** (the **first part** of the tariff) for the right to participate in the transaction, and a **price per unit** of product (the **second part**) → e.g. amusement/ theme parks, cover charge in clubs.
  - **Quantity Discount →** price discount for large purchases → e.g. buy 2 get 1 free scheme.
  - **Tie-in-Sale →** a consumer can buy one product only if another product is also bought → e.g. coffee machine that requires a special coffee capsule from the company.
  - **Quality Discrimination →** selling different qualities to different type of consumers.

First Degree Price Discrimination

- A monopolist can charge maximum price that each consumer is willing to pay → extracts ***all* consumer surplus**.
- Profit = total surplus → first-degree price discrimination is *efficient*.





First Degree Price Discrimination …

- First-degree price discrimination is **highly profitable <u>but</u>** requires <u>detailed information</u> and ability to <u>avoid arbitrage</u>.
- It leads to the **efficient choice of output**.
- **But** there are **other pricing schemes** that will **achieve the same outcome**:
  - **Non-Linear Prices** (e.g. **two-part pricing**, in which a lump-sum fee (membership fee) and a per unit price are charged.
  - **Block Pricing →** bundle total charge and quantity in a package.

Two-Part Pricing

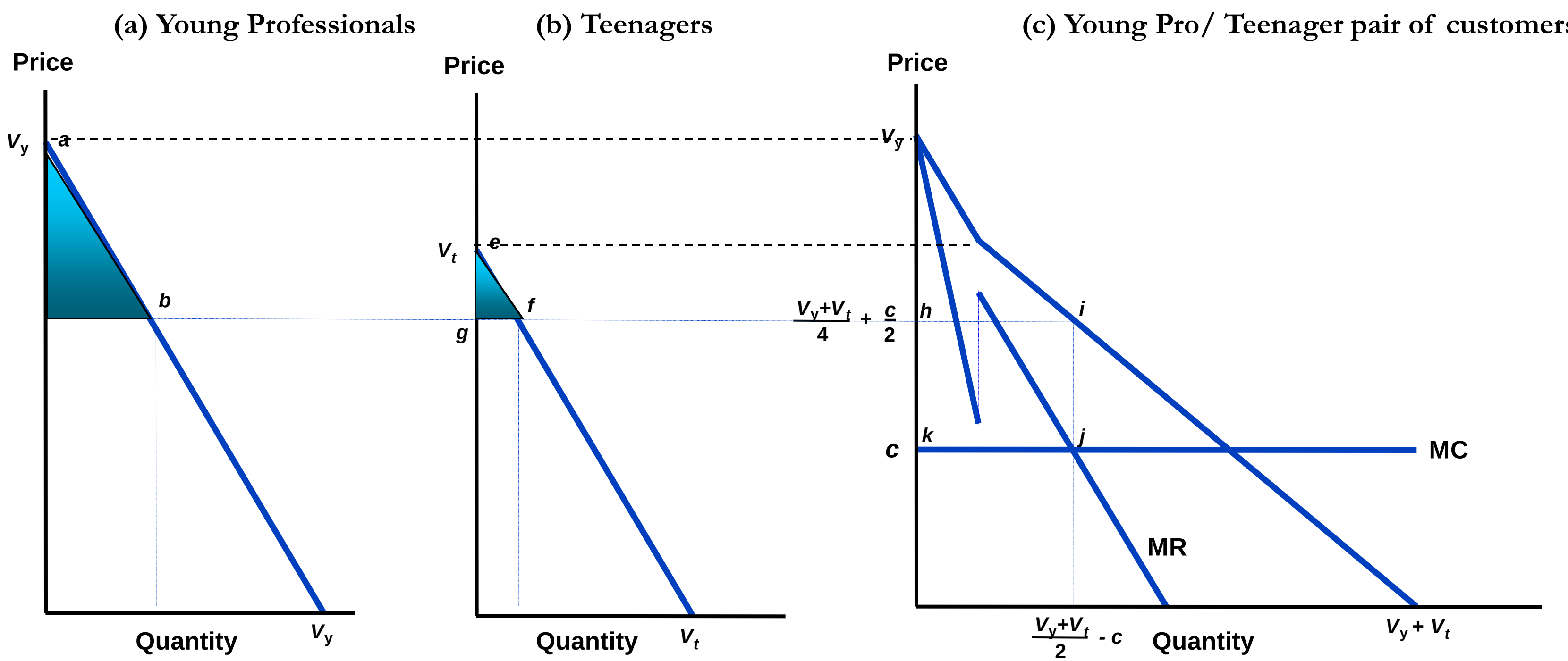
- Consider the following example → St. James Power Station Club → serves **two types of customers**; teenagers (t) and young professionals (p).
  - Young professionals → The demand for entry plus Q_y drinks is $P = V_y - Q_y$
  - Teenagers → The demand for entry plus Q_t drinks is $P = V_t - Q_t$
  - There are equal numbers of each type.
  - Assume that $V_y > V_t$ → young professionals are willing to pay more than teenagers.
  - Cost of operation of the club: $C(Q) = F + cQ$
- The demand and costs are all expressed in daily units

Two-Part Pricing …

- Suppose that the club owner applies "traditional" **linear pricing** → free entry (<u>no cover charge</u>) and <u>a set price</u> for drinks.
  - The aggregate demand is $Q_U = Q_y + Q_t = (V_y + V_t) - 2\,P_U$
  - The inverse demand is $P_U = (V_y + V_t)/2 - Q_U/2$
  - MR → $MR = (V_y + V_t)/2 - Q_U$
  - MR = MC, MC = c and solve for Q to give $Q_U = (V_y + V_t)/2 - c$
  - Substitute into the aggregate demand to give the equilibrium price $P_U = (V_y + V_t)/4 + c/2$
  - Each young professional buys $Q_y = (3V_y - V_t)/4 - c/2$ drinks.
  - Each teenager buys $Q_y = (3V_y - V_t)/4 - c/2$ drinks.
  - Profit from each pair of the young pro. and teenager is $\pi_U = (V_y + V_t - 2c)^2$

Two-Part Pricing …

This example can be illustrated as follows (the demand from each customer):



Linear pricing leaves each type of consumer with consumer surplus

Two-Part Pricing …

- If there are *n* customers of each type per night,

$$\Pi_U = n\pi_U - F = n(P_U - c)Q_U - F = \frac{n}{8}(V_y + V_t - 2c)^2 - F$$

- For example if **V_y=$16, V_t=$12, c=$4, n_y=100,** and **n_t=100** then,

$$P_U = \frac{(V_y + V_t)}{4} + \frac{c}{2} = \$9 \qquad Q_U = \frac{(V_y + V_t)}{2} - c = 10 \text{ drinks}$$

$$Q_y = \frac{(3V_y - V_t)}{4} - \frac{c}{2} = 7 \text{ drinks} \qquad Q_t = \frac{(3V_t - V_y)}{4} - \frac{c}{2} = 3 \text{ drinks}$$

$$\pi_U = \frac{1}{8}(V_y + V_t - 2c)^2 = \$50$$

$$\Pi_U = \frac{n}{8}(V_y + V_t - 2c)^2 - F = \$5000 - F \text{ each night}$$

Two-Part Pricing …

- The club owner can actually do better than just setting a uniform price.
- **Consumer surplus** at the uniform linear price:

$$CS_y^U = \frac{1}{2}\left(V_y - P_U\right)Q_y = \frac{1}{2}\left(Q_y\right)^2 = \frac{1}{2}\left(\frac{3V_y - V_t}{4} - \frac{c}{2}\right)^2 \quad \text{for young pro.}$$

$$CS_t^U = \frac{1}{2}\left(V_t - P_U\right)Q_t = \frac{1}{2}\left(Q_y\right)^2 = \frac{1}{2}\left(\frac{3V_t - V_y}{4} - \frac{c}{2}\right)^2 \quad \text{for teenager}$$

$$CS_y^U = \$24.5 \quad \text{for young pro.}$$

$$CS_t^U = \$4.5 \quad \text{for teenager.}$$

- These **CS** represents the **surplus that the monopolist fails to extract**. The firm can do better by setting a **two-part tariff**.

Two-Part Pricing …

- Charge the **young professionals** a **cover charge (E_y) equals to CS^U_y** and **teenagers** a **cover charge (E_t) equals to CS^U_t** → how to implement? → e.g. check ID on the front door.

$$E_y = \frac{1}{2}\left(\frac{3V_y - V_t}{4} - \frac{c}{2}\right)^2 \text{ for young pro. \& } E_t = \frac{1}{2}\left(\frac{3V_t - V_y}{4} - \frac{c}{2}\right)^2 \text{ for teenager}$$

- **Also, continue to charge the uniform pricing P_U per drink.**

- In our example: **E_y=\$24.5; E_t=\$4.5 and P_u=\$9**. Paying cover charge reduce the customers' surplus to zero but does not make it negative.

- This pricing will increase profit by Ey=\$24.5 per young professional and E_t=\$4.5 per teenager in addition to;

$$Q_y\left(P_U - c\right) = 7\left(\$9 - \$4\right) = \$35$$

$$Q_t\left(P_U - c\right) = 3\left(\$9 - \$4\right) = \$15$$

$$\Pi = n\left(\underbrace{\left(\$35 + E_y\right)}_{\pi_y} + \underbrace{\left(\$15 + E_y\right)}_{\pi_t}\right) - F$$

$$\Pi = 100\left(\$59.5 + \$19.5\right) = \$7900 - F$$

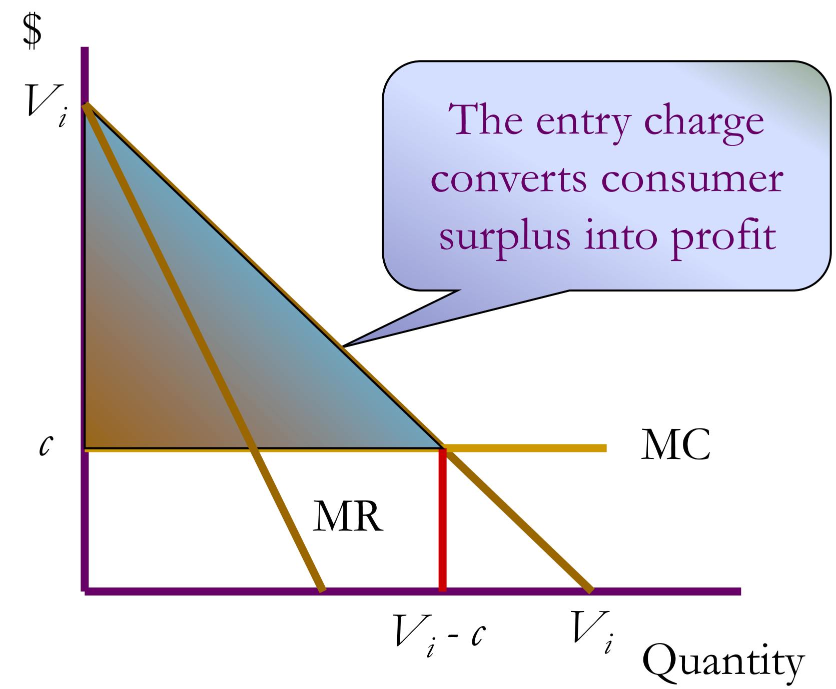
Two-Part Pricing …

- The club can **do even better** by:
  - **Reduce the price per drink** further below \$9 → customers enjoy some surplus → the club can extract this additional surplus by **increasing the cover charge**.

Set the unit price equal to marginal cost

This gives consumer surplus of $(V_i - c)^2/2$

Set the entry charge to $(V_i - c)^2/2$



Profit from each pair of a Young Pro. and a Teenager is now $\pi_d = [(V_y - c)^2 + (V_t - c)^2]/2$

Two-Part Pricing …

- Thus charge **P=MC=4**, and the cover charges are:

$$E_y = CS_y^U = \frac{1}{2}(V_y - c)Q_y \quad \text{for young pro.}$$

$$= \frac{1}{2}(V_y - c)^2 = \frac{1}{2}(\$16 - \$4)^2 = \$72 > \$59.5$$

$$E_t = CS_t^U = \frac{1}{2}(V_t - c)Q_t \quad \text{for teenager}$$

$$= \frac{1}{2}(V_t - c)^2 = \frac{1}{2}(\$12 - \$4)^2 = \$32 > \$19.5$$

- Thus, we have:

$$Q_y(P_U - c) = 12(\$4 - \$4) = \$0$$

$$Q_t(P_U - c) = 8(\$4 - \$4) = \$0$$

$$\Pi = n\left(\underbrace{(\$0 + E_y)}_{\pi_y} + \underbrace{(\$0 + E_y)}_{\pi_t}\right) - F$$

$$\Pi = 100(\$72 + \$32) - F = \$10400 - F$$

- The ability to practice **first-degree price discrimination** induces the monopoly to produce the **efficient** quantity → however, the **total surplus is gained solely by the monopolist**.

Block Pricing

- There is another pricing method that the owner can apply → **offer a <u>package</u> of "Entry plus X drinks for $Y"**.

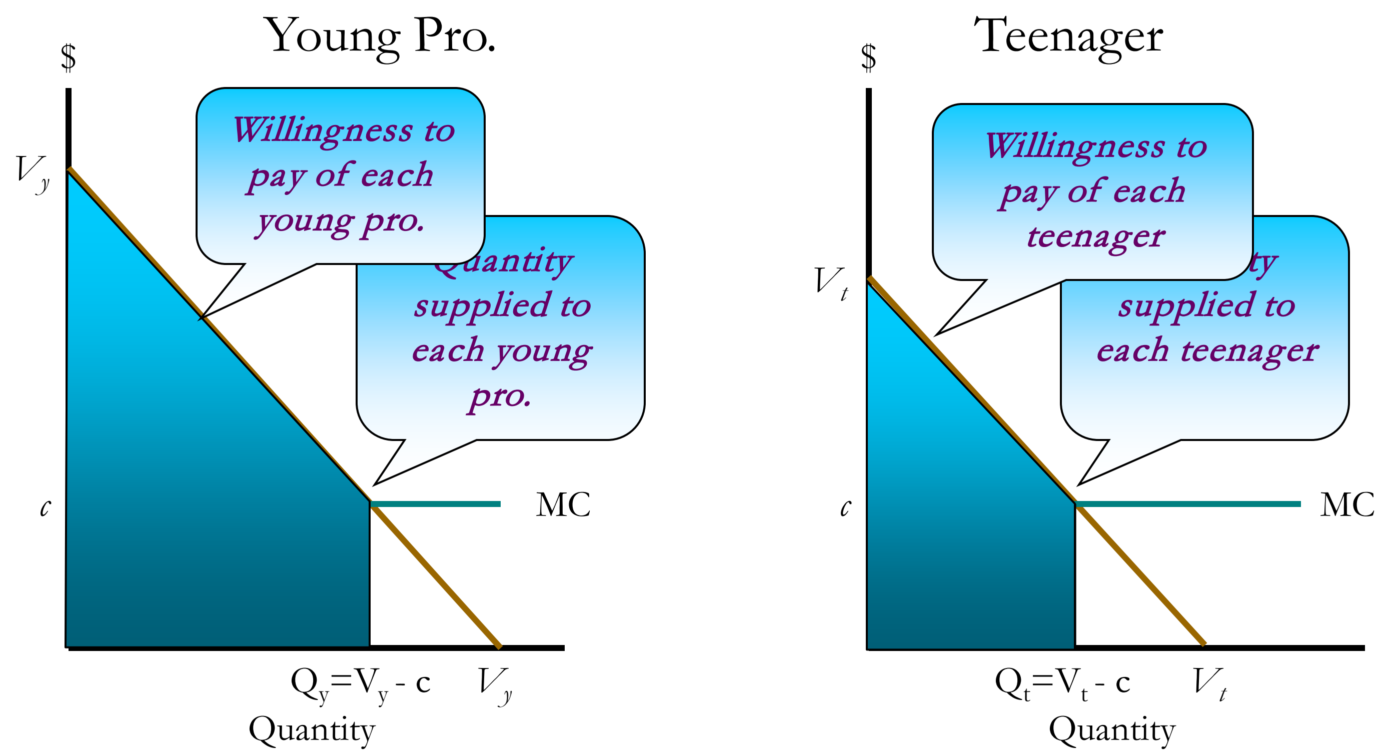
- To maximize profit apply the following rules:

  - Set the **<u>quantity</u>** offered to each costumer type **equal to the amount that type would buy** at **price equal to marginal cost (12 drinks & 8 drinks respectively)**.

  - Set the **<u>total charge</u>** for **each consumer type** to the **<u>total willingness to pay</u>** for the relevant quantity.

- <u>For example</u>: **<u>Entry and X amount of drinks for a price of Y.</u>**

Block Pricing …



$WTP_y = (V_y - c)^2/2 + (V_y - c)c = (V_y^2 - c^2)/2 = (16^2 - 4^2)/2 =$ **\$120**.

$WTP_t = (V_t - c)^2/2 + (V_t - c)c = (V_t^2 - c^2)/2 = (12^2 - 4^2)/2 =$ **\$64.**

Block Pricing …

- How to implement it?
  - Give customers a card at the entrance and give each of them the appropriate number of tokens (12 for the young pro. And 8 for the teenager) that can be exchanged with drinks at no additional charge.
- Profit from a consumer type *i* is the **fee equals to WTP minus the cost of the drinks**.

$$\pi_y = \frac{\left(V_y^2 - c^2\right)}{2} - c\left(V_y - c\right) = \frac{\left(V_y - c\right)^2}{2} = \$72 \text{ for young pro.}$$

$$\pi_t = \frac{\left(V_t^2 - c^2\right)}{2} - c\left(V_t - c\right) = \frac{\left(V_t - c\right)^2}{2} = \$32 \text{ for teenager}$$

- Profits are exactly the same as those obtained under the two-part pricing.
- Important conditions: → **1)** the club **can distinguish different type of consumers, 2)** the club can **deny entry to those do not want to pay**.

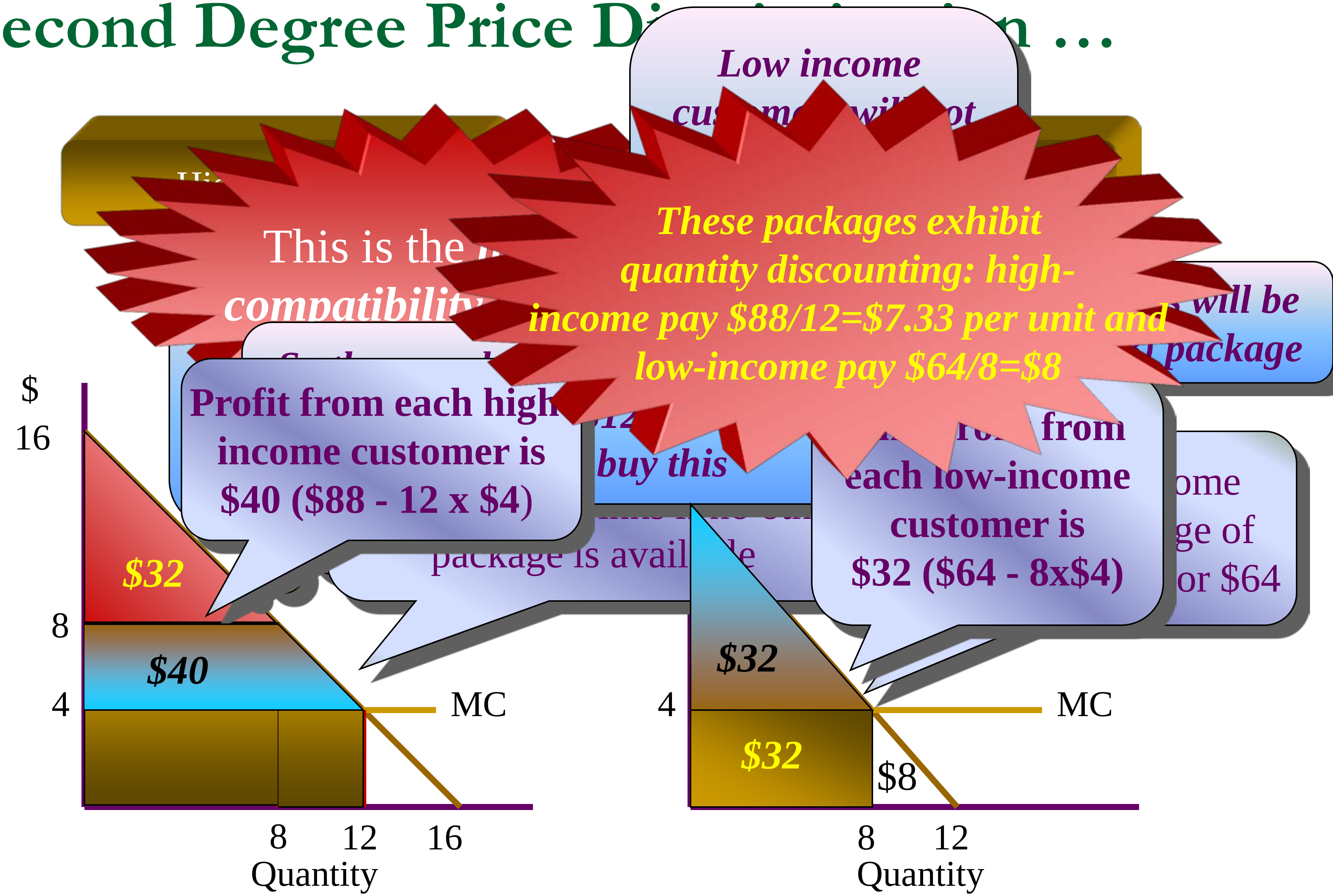
Second Degree Price Discrimination

- If the seller cannot distinguish the buyers' type, e.g. the WTP (income) → <u>asymmetric information</u> → the price discrimination based on the personalized pricing cannot be done.
- **A high income (WTP) buyer may pretend to be a low income buyer → to avoid having to pay a higher price**.
- **<u>Example</u>: P_H=16-Q_H and P_L=12-Q_L, and MC=$4**.
- Recall from the **First-Degree Price Discrimination**:
  - With **Two-Part-Pricing** → Charge an **entry fee of $72 for the high WTP (high income)** customers, and **$32 for the low WTP (low income)** customers, and set **P=MC=$4** per drink for both types.
  - With **Block Pricing** → Charge $120 **for entry plus (=WTP)** 12 drinks **to high income** customers and charge $64 **for entry plus (=WTP)** 8 drinks **to low income** customers.

Second Degree Price Discrimination …

- When the club **cannot distinguish the types of buyers (asymmetric information)**, the **first-degree price discrimination will not work**:
  - **High income** customers get **no surplus** from **the package price designed for them**, **BUT** can get **positive surplus** from **the package price designed for the other type**.
  - So they will **pretend to be low income customers** to be better off, although this means that they get only **smaller amount of drinks**.
- The pricing scheme has to be designed such that **each type of customers must prefer to choose the package designed for them** to the other package → it is incentive compatible → **menu pricing**.
- Such **menu pricing** should be designed such that:
  - Induce customers to **reveal their true types**.
  - **Self-select** the package designed for them.

Second Degree Price Di… n …



Second Degree Price Discrimination …

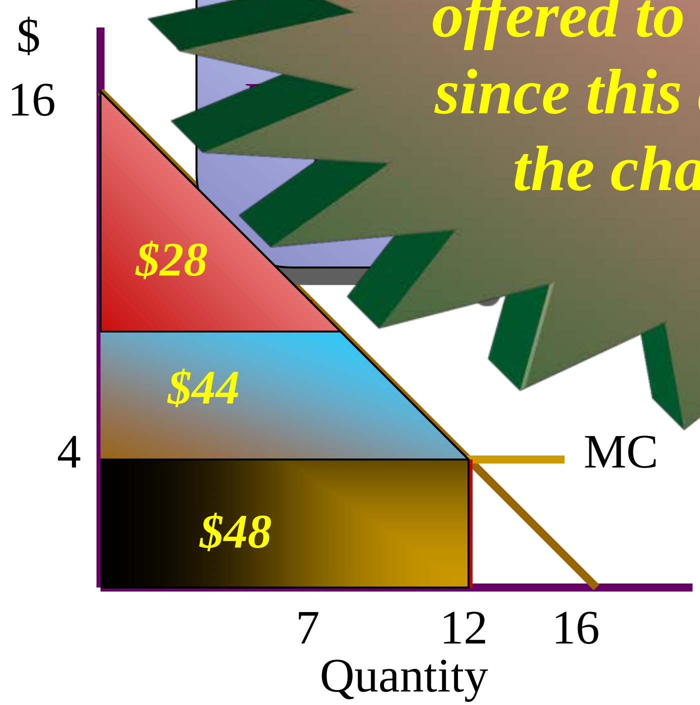
- **Incentive Compatibility Constraint**:
  - Any offer made to high demand customers must offer them as much consumer surplus as they would get from an offer designed for low-demand consumers.
- Thus, if the offer is incentive compatible, the high income customers **will never** take the package for the low income customers → price discrimination works even if you face asymmetric information.

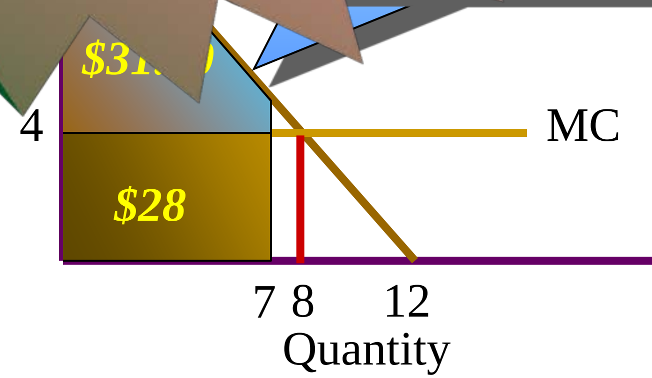
Second Degree Price Discrimination …

A high-incon... up to $8...

...low-income
...7 drinks
...will pay up to
...and 7 drinks
...each ($59.50, 7)
...50: a reduction
...50 per customer

The monopolist does better by reducing the number of units offered to low-income customers since this allows him to increase the charge to high-income customers





Second Degree Price Discrimination …

- **What is the optimal menu pricing then?**:
  - Keep reducing the quantity of drinks offered to low-income → make the **effective price higher** for them → unfortunately this will **decrease the profit from low income**.
  - But, the good news is, this will allow us to give a **smaller price reduction** (make sure that it is **still incentive compatible!**) to high income → **increase the profit from high income**.
  - Keep doing that until the reduction in profit from the low income = to the increase in profit from the high income.
- Trade-off: **Informational Rents** vs. **Efficiency**.

Second Degree Price Discrimination …

- Will the monopolist always want to **supply both types** of consumer?
- Not always → there are cases where it is **better to supply only high-demand types** → high-class restaurants, golf and country clubs.
- Take our example again;
  - Suppose that there are $\mathbf{N_l}$ low-income consumers and $\mathbf{N_h}$ high-income consumers.
  - Suppose both types of customers are served → then the club offers **($ 57.5 ; 7 drinks)** for the low income customers and **($ 92 ; 12 drinks)**.
  - Profit will be: $\Pi = \$31.5(N_l) + \$44(N_h)$
  - Suppose now only high income customers are served → the club can set the package **($ 120 ; 12 drinks),** and profit will be $\Pi = \$72(N_h)$

Second Degree Price Discrimination …

- Serving both types of customers is profitable if and only if:

$$\$31.5(N_l)+\$44(N_h)>\$72(N_h) \quad \rightarrow \quad \$31.5(N_l)+28(N_h)$$

$$\frac{N_h}{N_l}<\frac{\$31.5}{\$28}=1.125$$

- Serving both types is profitable as long the proportion of high type consumers is not too large.

- **Characteristics of the second degree price discrimination**.
  - **Extract all consumer surplus** from the **low income type** group.
  - Leave **some consumer surplus** to the **high income type** who has incentive to pretend to be the low income type → because of the **informational rents**.
  - Offer **less than the socially efficient quantity** to **the low income type but give the socially efficient quantity to the high type group**.

Third Degree Price Discrimination

- Consumers differ by some **observable characteristic**(s).
- A uniform price is charged to all consumers in a particular group – linear price.
- Different uniform prices are charged to different groups, for examples:
  - "Kids are free"
  - "Subscriptions to professional magazines → different fee schedule.
  - Supermarket discount using clip on coupons.
  - Early-bird specials or happy hours; first-runs of movies vs. video.
- The pricing rule is:
  - Consumers with **low elasticity of demand** should be charged **high price**, and those with **high elasticity of demand** should be charged a **low price**.

Third Degree Price Discrimination …

- An example: The latest Harry Porter book → sold in the US and Europe.
- The demand in the US: $P_U=36 - 4Q_U$ and the demand in Europe: $P_E=24 - 4Q_E$, MC=\$4.
- Suppose that a **<u>uniform price</u>** is charged in both the US and Europe.
  - Invert the demand in the U.S.

$$P_U = 36 - 4Q_U \quad \rightarrow \quad Q_U = 9 - \frac{1}{4}P_U \ \text{ for } \ P_U \leq 36$$

$$P_E = 24 - 4Q_E \quad \rightarrow \quad Q_E = 6 -$$

  - Derive the aggregate demand.

$$\text{denote } P_U = P_E = P$$

$$Q = Q_U + Q_E = 9 - \frac{1}{4}P \ \text{ for } 36 \leq P \leq$$

$$Q = Q_U + Q_E = 15 - \frac{1}{2}P \ \text{ for } 0 \leq P < 24$$

At these prices only the US market is active

Now both markets are active

Third Degree Price Discrimination …

- Suppose that a **uniform price** is charged in both the US and Europe (Continued…)
  - Invert the direct demands:

$$Q = Q_U + Q_E = 9 - \frac{1}{4}P \text{ for } 24 \le P \le 36$$

$$P = 36 - 4Q \quad \text{for} \quad 0 \le Q \le 3$$

$$Q = Q_U + Q_E = 15 - \frac{1}{2}P \text{ for } 0 \le P < 24$$

$$P = 30 - 2Q \quad \text{for} \quad 3 \le Q \le 15$$

  - Marginal revenue:

$$MR = 36 - 8Q \quad \text{for} \quad 0 \le Q \le 3$$

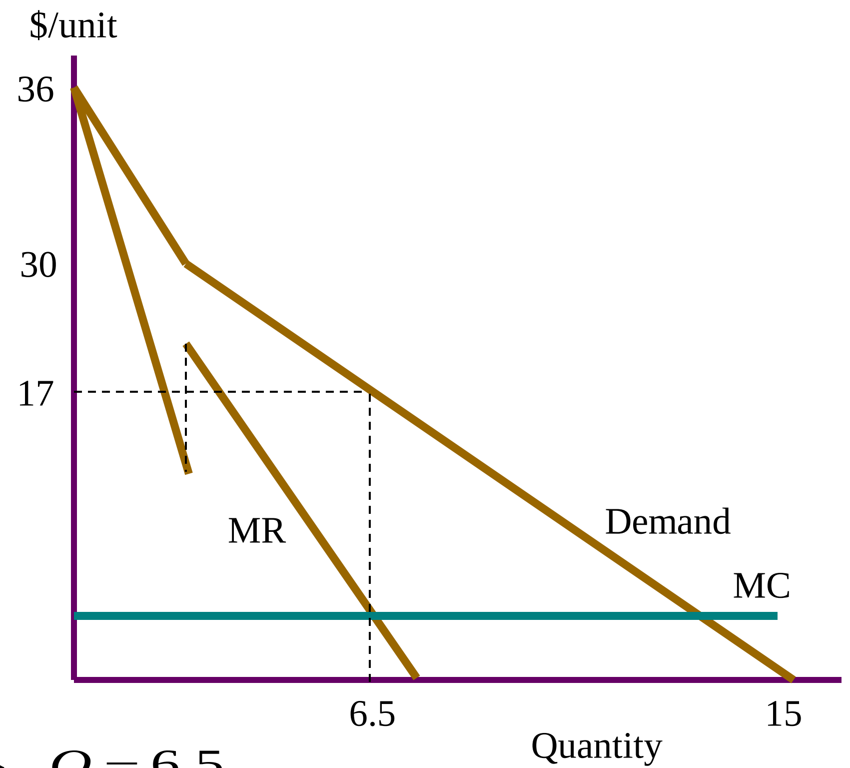
$$MR = 30 - 4Q \quad \text{for} \quad 3 \le Q \le 15$$

  - Profit maximization:

$$MR = MC \quad \rightarrow \quad 30 - 4Q = 4 \quad \rightarrow \quad Q = 6.5$$

  - Eq. price:

$$P = \$17$$



Third Degree Price Discrimination …

- Suppose that a **uniform price** is charged in both the US and Europe (Continued…)
  - Substitute the eq. price into the individual demand functions:

$$Q_U = 9 - \frac{1}{4}P_U = 9 - \frac{1}{4}(17) = 4.75\ million$$

$$Q_E = 6 - \frac{1}{4}P_E = 6 - \frac{1}{4}(17) = 1.75\ million$$

  - **Aggregate profit**:

$$\Pi = (P_U - c)Q = (17 - 4)(6.5) = \$84.5\ million$$

  - The firm can do better than this → notice that the MR is not equal to MC in both markets (when we look at each individual market) → MR>MC in Europe and MR<MC in the U.S.
  - What if different prices be charged in different markets **(price discrimination)**?
  - Consider each market separately → set in each market MR=MC → get the eq. price in each market.

Third Degree Price Discrimination …

Demand in the US:

$P_U = 36 - 4Q_U$

Marginal revenue:

$MR = 36 - 8Q_U$

$MC = 4$

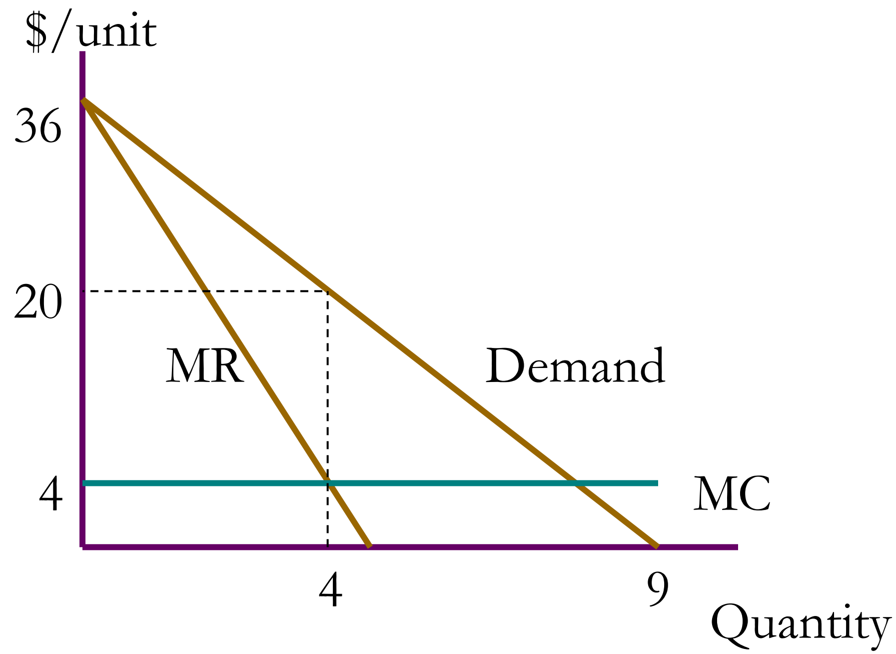
Equate MR and MC

$36 - 8Q_U = 4$

$Q_U = 4$

Price from the demand curve

$P_U = \$20$



Third Degree Price Discrimination …

Demand in the Europe:

$P_E = 24 - 4Q_U$

Marginal revenue:

$MR = 24 - 8Q_U$

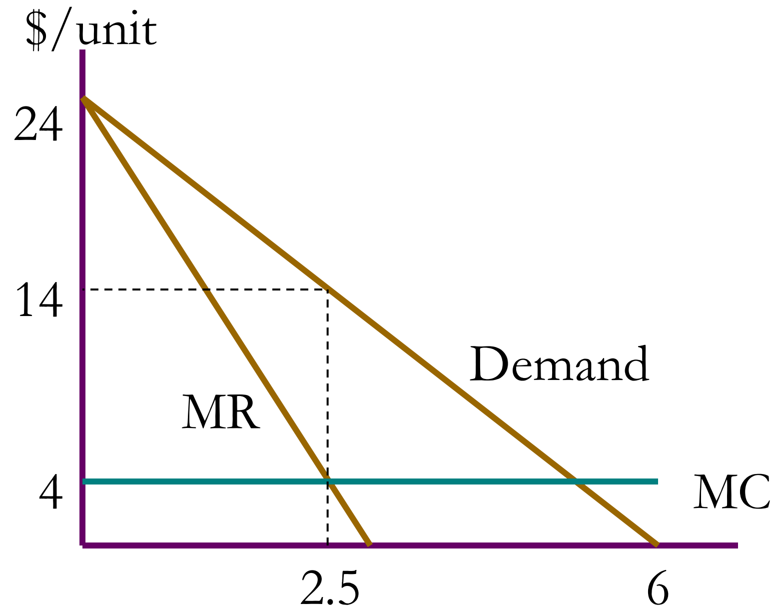
$MC = 4$

Equate MR and MC

$Q_E = 2.5$

Price from the demand curve

$P_E = \$14$



Aggregate sales are 6.5 million books → the same as without price disc, hence:

$$\Pi = \pi_U + \pi_E = (20-4)4 + (14-4)2.5 = \$89\ millions$$

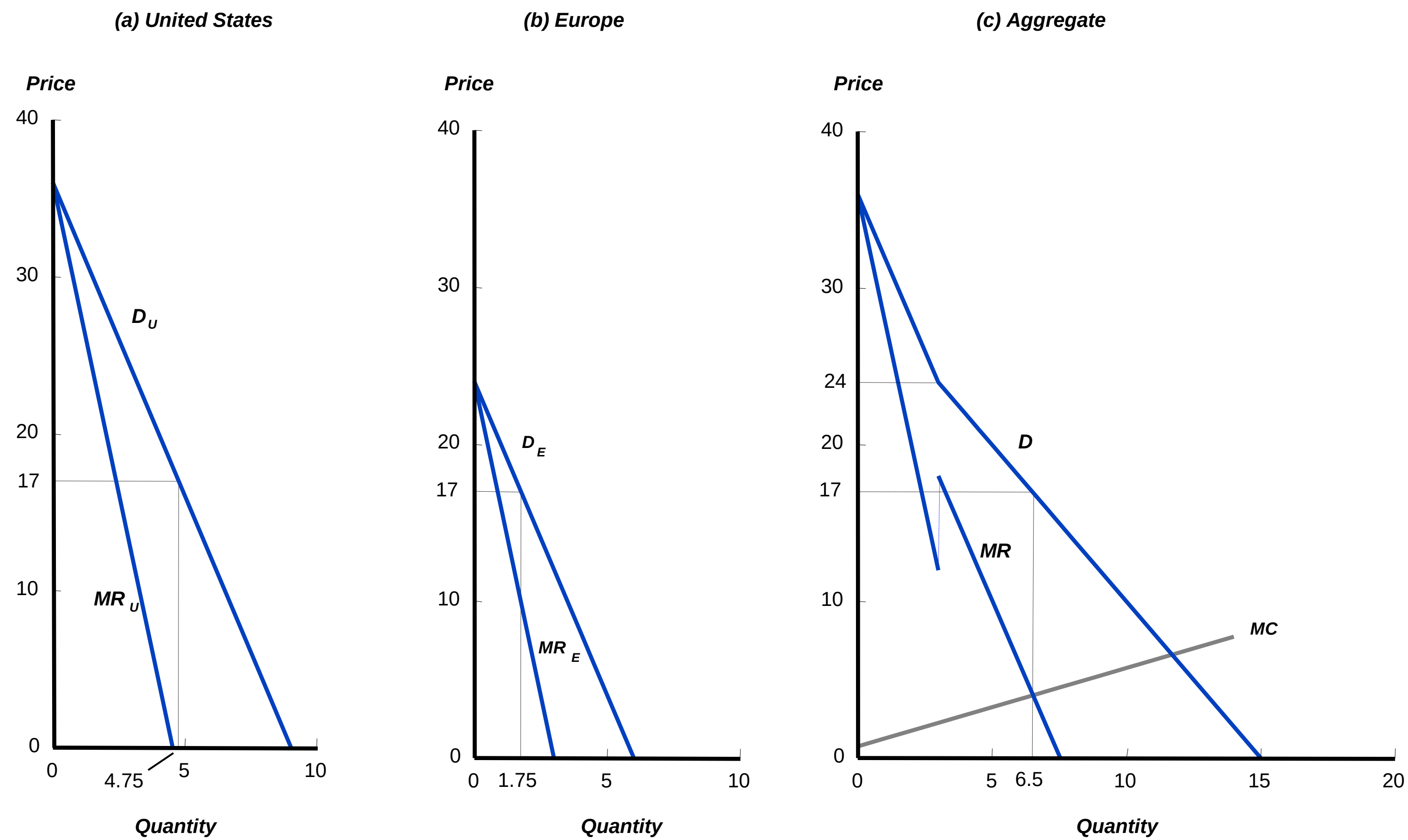
We have **\$4.5 million greater than without price discrimination**.

Third Degree Price Discrimination …

- What if **MC is not constant** but instead is **increasing**? → e.g. MC is increasing → $MC = 0.75 + \frac{1}{2}Q$
- Similar to what we'd done before, we can derive the **solutions** under **<u>no price discrimination</u>** (**uniform pricing**) by applying these steps (please redo and verify ☺):
  - Derive the aggregate demand as is done previously..
  - Derive the associated MR.. From our example, if Q>3 both markets are served → MR=30-4Q.
  - MR=MC → 0.75+Q/2=30-4Q → Q*=6.5 million books.
  - Derive the equilibrium **uniform price** → since both markets are active, the relevant part of the aggregate demand fu. is P=30-2Q → for Q*=6.5 we have P*=$17.
  - Calculate the demand in each market → 4.75 million books in the US and 1.75 million books in Europe → get the resulting profit.

Third Degree Price Discrimination …

- Graphical depiction:

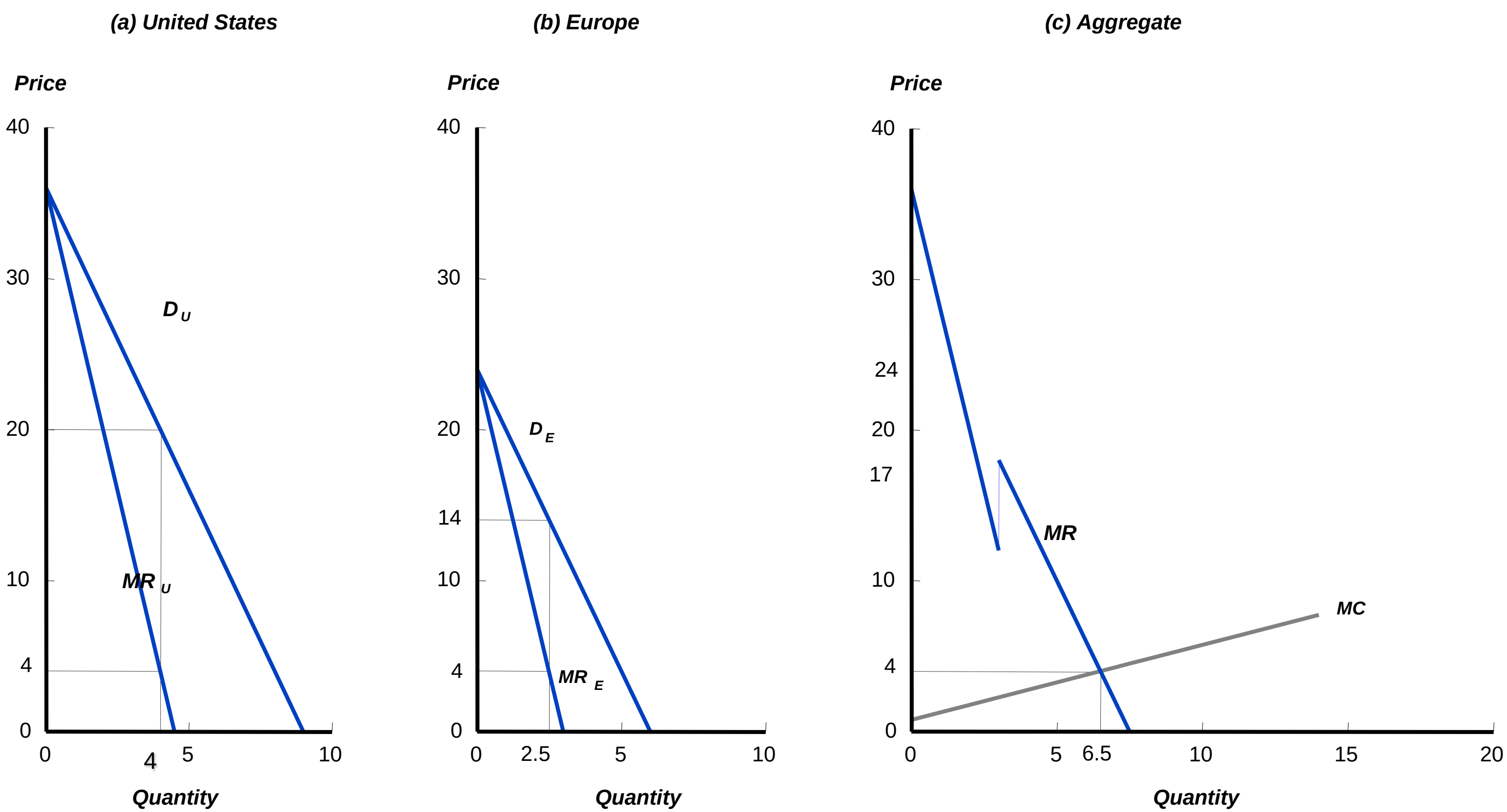


Third Degree Price Discrimination …

- Solutions with **price discrimination** can be derived by applying these steps (please redo and verify ☺):
  - Derive the **MR in each market** and sum-up these MRs to get the **aggregate MR** →
    - $MR=36-8Q_u$ for P<$36 and $MR=24-8Q_E$ for P<$24 → Inverting these MRs we have; $Q_u=4.5-MR/8$ and $Q_E=3-MR/8$ → summing these yield the aggregate MR.
      - $Q=Q_u+Q_E=4.5-MR/8$ for $Q \leq 3$ → or $MR=36-8Q$ for $Q \leq 3$
      - $Q=Q_u+Q_E=7.5-MR/4$ for Q>3. → or $MR=30-4Q$ for Q>3
  - **MR=MC** → to get the **aggregate output** → $30-4Q=0.75+Q/2$ → $Q^*=6.5$ million books → $MR=30-4(6.5)=$**$4** (this is the **equilibrium MR and also MC**).
  - Identify **equilibrium quantities in each market** by equating the MR in each market from the aggregate MR curve → In the US: $36-8Q_u=4$ or **$Q^*_u=4$ million books** → In Europe: $24-8Q_u=4$ or **$Q^*_E=2.5$ million books**.
  - Identify **equilibrium prices in each market** from individual market demands → $P^*_u=36-4Q^*_u=36-4(4)=$**$20** in the US and $P^*_E=24-4Q^*_E=24-4(2.5)=$**$14** in Europe.

Third Degree Price Discrimination …

- Graphical depiction:



Third Degree Price Discrimination …

- Often arises when firms sell <u>differentiated products</u>, for examples:
  - Hard-back versus paper back books
  - First-class versus economy airfare
- The seller needs an easily observable characteristic that signals willingness to pay.
- The seller must be able to **<u>prevent arbitrage</u>**:
  - An airline company can require a Saturday night stay for a cheap flight.
  - Time of purchase of movie ticket.
  - Requiring proof (student ID card).
  - Provide rebate coupons on the newspapers.

Third Degree Price Discrimination …

- A more **general** recipe of deriving the discriminatory and uniform pricing rules.
  - Suppose a monopolist supplies 2 groups of consumers with the following inverse demands.

$$P_1 = A_1 - B_1 Q_1 \quad \text{and} \quad P_2 = A_2 - B_2 Q_2 \quad \text{assume } A_1 > A_2$$

  - Inverting the inverse demands:

$$Q_1 = \frac{(A_1 - P)}{B_1} \quad \text{and} \quad Q_2 = \frac{(A_2 - P)}{B_2}$$

  - Thus, the aggregate demand is:

$$Q = Q_1 + Q_2 = \frac{A_1 B_2 + A_2 B_1}{B_1 B_2} - \frac{B_1 + B_2}{B_1 B_2} P$$

$$\text{this holds for } P < A_2$$

  - The MR associated with the above aggregate demand:

$$MR = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} - 2\frac{B_1 B_2}{B_1 + B_2} Q$$

Third Degree Price Discrimination …

- A more **general** recipe of deriving the discriminatory and uniform pricing rules …
  - Suppose for simplicity assume MC=0 (this can of course be relaxed), hence the profit max. condition requires MR=MC → MR=0 → solve for Q^*_U.

$$Q_U^* = \frac{A_1 B_2 + A_2 B_1}{2 B_1 B_2}$$ **under the uniform pricing rule.**

  - Substituting Q^*_U into the aggregate inverse demand yields.

$$P_U^* = \frac{A_1 B_2 + A_2 B_1}{2(B_1 + B_2)}$$

  - Substituting P^*_U into the individual demands gives the equilibrium output in each market.

$$Q_{U_1}^* = \frac{(2A_1 - A_2)B_1 + A_1 B_2}{2B_1(B_1 + B_2)} \text{ and } Q_{U_2}^* = \frac{(2A_2 - A_1)B_2 + A_2 B_1}{2B_2(B_1 + B_2)}$$

Third Degree Price Discrimination …

- A more **general** recipe of deriving the discriminatory and uniform pricing rules …
  - Under the **discriminatory pricing rule**, the firm sets MR=MC for each group. We know that MRs are:

$$P_1 = A_1 - B_1Q_1 \quad \text{and} \quad P_2 = A_2 - B_2Q_2$$

$$TR_1 = (A_1 - B_1Q_1)Q_1 \quad \text{and} \quad TR_2 = (A_2 - B_2Q_2)Q_2$$

$$MR_1 = A_1 - 2B_1Q_1 \quad \text{and} \quad MR_2 = A_2 - 2B_2Q_2$$

  - The equilibrium outputs and prices for each group:

$$MR_1 = MC \;\rightarrow\; A_1 - 2B_1Q_1 = 0 \;\rightarrow\; Q^*_{D_1} = \frac{A_1}{2B_1}$$

$$MR_2 = MC \;\rightarrow\; A_2 - 2B_2Q_2 = 0 \;\rightarrow\; Q^*_{D_2} = \frac{A_2}{2B_2}$$

$$P^*_{D_1} = A_1 - B_1Q^*_{D_1} = \frac{A_1}{2} \quad \text{and} \quad P^*_{D_2} = A_2 - B_2Q^*_{D_2} = \frac{A_2}{2}$$

  - We have: $Q^*_{D_1} < Q^*_{U_1}$ and $Q^*_{D_2} > Q^*_{U_1}$ and $Q^*_{D_1} + Q^*_{D_2} = Q^*_U$

Third Degree Price Discrimination …

- We can also verify:

$$MR_1 = P_1 + \frac{\partial P_1}{\partial Q_1} Q_1 = P_1 \left(1 + \frac{\partial P_1}{\partial Q_1} \frac{Q_1}{P_1} \right)$$

$$MR = P_1 \left(1 + \frac{1}{\varepsilon_1} \right) \quad \text{with} \quad \varepsilon_1 = \frac{\partial Q_1}{\partial P_1} \frac{P_1}{Q_1}$$

$$\text{Similarly} \quad MR_2 = P_2 \left(1 + \frac{1}{\varepsilon_2} \right)$$

- With price discrimination: $MR_1 = MR_2$, and thus;

$$P_1 \left(1 + \frac{1}{\varepsilon_1} \right) = P_2 \left(1 + \frac{1}{\varepsilon_2} \right) \quad \text{or} \quad \frac{P_1}{P_2} = \frac{1 + \frac{1}{\varepsilon_2}}{1 + \frac{1}{\varepsilon_1}} = \frac{\varepsilon_1 \varepsilon_2 + \varepsilon_1}{\varepsilon_1 \varepsilon_2 + \varepsilon_2}$$

- If the demand curve is elastic → $\varepsilon < -1$ and if the demand curve is inelastic → $-1 < \varepsilon < 0$. Thus, **price will be lower in the market with higher elasticity of demand**.